

HEURISTIC APPROACH FOR BICRITERIA IN TWO STAGE OPEN SHOP SCHEDULING PROBLEM

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ABSTRACT

This Paper is an attempt to obtain an optimal solution for minimizing the bicriteria so as to minimize the total rental cost of the machines subject to obtain the minimum makespan for n-jobs 2 machines open shop scheduling problem in which the processing times of machines are given. Numerical illustration is given to justify the proposed algorithm.

Keywords: Open shop scheduling, Rental policy, Processing time, Latest time, Utilization time, elapsed time.

1. INTRODUCTION

Scheduling is one of the optimization problem found in real industrial content for which several heuristic procedures have been successfully applied. Scheduling is a form of decision making that plays a crucial role in manufacturing and service industries. It deals with allocation of resources to tasks over given time periods and its goal is to optimize one or more objectives. Flow shop scheduling problem has been one of the classical problem in production scheduling since Johnson [12] proposed the well known Johnson's rule in the two stage flow shop makespan scheduling problem. The work was developed by Jackson J.R [13], Smith [16], Maggu and Das [15], Yoshida and Hitomi [17], D.Rebaine [6], Chandasekharan [4], Anup [1] and Gupta Deepak [8] by considering various parameters such as Transportation time, Breakdown Interval, setup time, Weightage etc. Open shop scheduling differ from flowshop scheduling in the sense that there are no restriction placed on the order of the machines i.e, operations can be performed in any order first machine to second machine or second machine to first machine and not known in the advances. Hence the order of the machines can be selected arbitrarily. Maggu P.L and Harbans lal [14] introduced the concept of $n \times 2$ Open shop scheduling problem including job-block criteria. Gupta Deepak and Singh T.P [7] have studied two stage Open shop scheduling problem to minimizing the idle time of the machines in

which processing time are associated with their respective probabilities including job block criteria. further work was extended by gupta Deepak and renuka [11] by associated transportation time and weightage of jobs.

Recently Scheduling, so as to approximate more than one criterion received considerable attention. The bicriteria scheduling problems are motivated by the fact that they are meaningful from practical point of view. The bicriteria scheduling problems are generally divided into three classes. In the first class, the problem involved minimizing one criterion subject to the constraint that the other criterion to be optimized. In the second class, both criteria are considered equally important and problem involves finding efficient schedules. In third class, both criteria are weighted differently and an objective function as the sum of the weighted function is defined. The problem considered in this paper belong to the first class. Chandrasekharan Rajendra [4] introduced the concept "two stage flowshop scheduling problem with bicriteria". This work was extended by Bagga P.C & Bhambani. A[3], Chakarvarthy K & Rajendra C [5], Gupta.D & Singh T.P[7], Gupta.D & Sharma.S [8] by considering various parameters. In this present paper we have developed a new heuristic algorithm which gives minimum possible rental cost while minimizing total elapsed time simultaneously in two stage open shop scheduling problem.

2. PRACTICAL SITUATION

Open shop scheduling problems arise in several industrial situations. For example, consider a large aircraft garage with specialized work-centers. An airplane may require repairs on its engine and electrical circuit system. These two tasks may be carried out in any order but it is not possible to do these tasks on the same plane simultaneously. Other applications of open shop scheduling problems are in automobile repair, quality control centers, semiconductor manufacturing, teacher-class assignments, examination scheduling, and satellite communications etc. In the era of globalization or global uncertainties, to meet the challenges of the business, one does not always have enough funds to invest in

advanced machines to update the technology. Under such circumstances the machines has to be taken on rent. Rental of machines is an affordable and quick solution for having the equipment and up gradation to new technology

3. NOTATION

- S :Sequence of jobs 1,2,3,.....,n
- M_j :Machine, j=1,2,3.....
- A_i :Processing time of ith job on machine A
- B_i :Processing time of ith job on machine B
- C_j :Rental cost per unit time of machine j
- CT(S_K) :Total completion time of the jobs for sequence S_K
- CT(S_{K'}) : Total completion time of the jobs for sequence S_{K'}
- L₂(S_K) :Latest time when second machine is taken on rent for sequence S_K
- L₂(S_{K'}) :Latest time when second machine is taken on rent for sequence S_{K'}
- U₂(S_K) :Utilization time of second machine for sequence S_K
- U₂(S_{K'}) :Utilization time of second machine for sequence S_{K'}
- R(S_K) :Rental cost for sequence S_K
- R(S_{K'}) :Rental cost for sequence S_{K'}

4. RENTAL POLICY

The machines will be taken on rent as and when they are required and are returned as and when they are no longer required.

5. PROBLEM FORMULATION

Let n jobs 1,2,.....,n be processed on two machines M₁ and M₂ in any order i.e. the jobs will be processed first on M₁ and then on M₂ or first on M₂ and then on M₁ under the specified rental policy . Let A_i be the processing time of ith job on machine M₁ and B_i be the processing time of ith job on machine M₂ Our aim is to find the optimal or near optimal sequence {S_K} of the jobs which minimizing the rental cost of machines.

Tableau-1

Jobs	Machine M ₁	Machine M ₂
i	A _i	B _i
1	A ₁	B ₁
2	A ₂	B ₂
3	A ₃	B ₃
-	-	-
n	A _n	B _n

Mathematically, the problem is stated as: Minimize

$$R(S_K) = \sum_{i=1}^n A_i(S_K) \times C_1 + U_2(S_K) \times C_2$$

Subject to constraint: Rental Policy(P).

Our objective is to minimize rental cost of machines while minimizing the utilization time.

6. ASSUMPTION

1. Two jobs cannot be processed on a single machine at a time.
2. Jobs are independent to each other.
3. Per-emption is not allowed i.e. once a job started on a machine, the process on that machine cannot be stopped unless the job is completed.
4. Let n jobs be processed through two machines M₁ and M₂ in order M₁→ M₂ and in order M₂→M₁.
5. Machine break down is not considered.

7. ALGORITHM

Step1:For order M₁→M₂

Construct a Set S_A of all processing time A_i where A_i ≤ B_i and Set S_{A'} of all processing time A_i where A_i ≥ B_i.

Step 2: Let S₁ denote a sub optimal sequence of jobs corresponding to non-decreasing time in set S_A ,Similarly S_{1'} corresponding to set S_{A'}.

Step 3: The augmented ordered sequence S_K = (S₁ ,S_{1'}) are optimal/or near optimal sequence for processing the jobs on machine M₁ which given the minimum rental cost for given problem.

Step 4: For order M₂→M₁

Construct a Set S_B of all processing time B_i where B_i ≤ A_i and Set S_{B'} of all processing time B_i where B_i ≥ A_i.

Step 5: Let S₂ denote a sub optimal sequence of jobs corresponding to non-decreasing time in set S_B ,Similarly S_{2'} corresponding to set S_{B'}.

Step 6: The augmented ordered sequence S_{K'} = (S₂ ,S_{2'}) are optimal/or near optimal sequence for processing the jobs on machine M₂ which given the minimum rental cost for given problem.

Step 7: Prepare in-out table of sequence S_K = (S₁ ,S_{1'}) and S_{K'} = (S₂ ,S_{2'}) in the order M₁→M₂ and M₂→M₁ respectively.

Step 8: Compute the total completion time CT(S_K) and CT(S_{K'}) by computing in-out table for sequence S_K and S_{K'} respectively.

Step 9: Calculate latest time at which second machine is taken on rent as follow:

$$L_2(S_K) = CT(S_K) - \sum_{i=1}^n B_i$$

$$L_2(S_{K'}) = CT(S_{K'}) - \sum_{i=1}^n A_i$$

Step 10: Prepare in-out table for sequence S_K and S_{K'} having latest time on second machine is L₂(S_K) and L₂(S_{K'}) respectively.

Step 11: Compute utilization time U₂(S_K) and U₂(S_{K'}) of second machine as follow:

$$U_2(S_K) = CT(S_K) - L_2(S_K)$$

$$U_2(S_{K'}) = CT(S_{K'}) - L_2(S_{K'})$$

Step 12: Find rental cost for sequence S_K and S_{K'} as follow:

$$R(S_K) = \sum_{i=1}^n A_i(S_K) \times C_1 + U_2(S_K) \times C_2$$

$$R(S_{K'}) = \sum_{i=1}^n B_i(S_{K'}) \times C_2 + U_2(S_{K'}) \times C_1$$

Where C₁ and C₂ are the rental cost per unit time of machine M₁ and M₂ respectively.

8. NUMERICAL ILLUSTRATION

Consider 5 jobs and 2 machines open shop problem to minimize the rental cost. The Processing time of these jobs are giveb as follow. Rental cost per unit time for machine M₁ and M₂ are 10 and 15 respectively.

Tableau-2

Jobs	Machine M ₁	Machine M ₂
i	A _i	B _i
1	22	18
2	28	30
3	32	16
4	17	21
5	20	25

Our objective is to obtain optimal schedule and order to minimize total elapsed subject to minimize the total rental cost of the machines, under the rental policy P.

SOLUTION:

For order M₁ → M₂

Tableau-3

Jobs	Machine M ₁	Machine M ₂
i	A _i	B _i
1	22	18
2	28	30
3	32	16
4	17	21
5	20	25

As per step 1:

Construct a set S_A and S_A'

$$S_A = \{ 28, 17, 20 \} \quad S_A' = \{ 22, 32 \}$$

As per step 2:

$$S_1 = \{ 4, 5, 2 \} \quad S_1' = \{ 1, 3 \}$$

As per step 3:

Augmented ordered sequence S_K = { 4, 5, 2, 1, 3 }

As per step 7:

In-out table of sequence S_K for order M₁ → M₂ as follow in table-4

Tableau-4

Jobs	Machine M ₁	Machine M ₂
4	0-17	17-38
5	17-37	38-63
2	37-65	65-95
1	65-87	95-113
3	87-119	119-135

As per step 8 and 9:

Total completion time for sequence S_K is CT(S_K) = 135
Latest time at which second machine is taken on rent for sequence S_K

$$L_2(S_K) = 135 - 110 = 25$$

As per step 10:

In-out table for sequence S_K having latest time on second machine is 25 as follow in Table-5

Tableau-5

Jobs	Machine M ₁	Machine M ₂
4	0-17	25-46
5	17-37	46-71
2	37-65	71-101
1	65-87	101-119
3	87-119	119-135

As per step 11 and 12:

Utilization time of second machine for sequence S_K

$$U_2(S_K) = 135 - 25 = 110$$

$$\text{Rental cost } R(S_K) = 119 \times 10 + 110 \times 15 = 2840$$

Total Elapsed time = 135

Order M₂ → M₁

Tableau-6

Jobs	Machine M ₂	Machine M ₁
i	B _i	A _i
1	18	22
2	30	28
3	16	32
4	21	17
5	25	20

As per step 4 and 5:

Construct a set S_B and S_B'

$$S_B = \{ 18, 16 \} \quad S_B' = \{ 30, 21, 25 \}$$

$$S_2 = \{ 3, 1 \} \quad S_2' = \{ 4, 5, 2 \}$$

As per step 6:

Augmented ordered sequence S_K' = { 3, 1, 4, 5, 2 }

As per step 7:

In-out table of sequence S_K' for order M₂ → M₁ as follow in table-7

Tableau-7

Jobs	Machine M ₂	Machine M ₁
3	0-16	16-48
1	16-34	48-70
4	34-55	70-87
5	55-80	87-107
2	80-110	110-138

As per step 8 and 9:

Total completion time for sequence S_K' is CT(S_K') = 138
Latest time at which second machine is taken on rent for sequence S_K'

$$L_2(S_K') = 138 - 119 = 19$$

As per step 10:

In-out table for sequence S_K' having latest time on second machine is 19 as follow in table-8

Tableau-8

Jobs	Machine M ₂	Machine M ₁
3	0-16	19-51
1	16-34	51-73
4	34-55	73-90
5	55-80	90-110
2	80-110	110-138

As per step 11 and 12:

Utilization time of second machine for sequence S_K'

$$U_2(S_K') = 138-19 = 119$$

$$\text{Rental cost } R(S_K') = 110 \times 15 + 119 \times 10 = 2840$$

Total elapsed time= 138

Total rental cost when the order is from M₁ to M₂ for the sequence (4,5,2,1,3) is 2840 units and total elapsed time=135 and when order M₂ to M₁ for sequence (3,1,4,5,2) rental is 2840 and total elapsed time is 138. Hence order M₁ to M₂ is optimal order which minimize total elapsed time and rental cost simultaneously.

9. REMARKS

1. Operations can be performed in any order first machine to second machine or second machine to first machine and not known in the advances.
2. The work may further be extended for n jobs 3 machines open shop problem.
3. The study can further be extended by considering various parameters such as transportation time, job-block, setup time, breakdown intervals etc

REFERENCES

1. Anup (2002), *“On two machine flow shop problem in which processing time assumes probabilities and there exists equivalent for an ordered job block”*, JISSOR, Vol. XXIII No. 1-4, pp. 41-44.
2. Bagga P C (1969), *“Sequencing in a rental situation”*, Journal of Canadian Operation Research Society, Vol.7, pp.152-15.
3. Bagga, P.C.& Bhambani, A.(1997), *“Bicriteria in flow shop scheduling problem”*, Journal of Combinatorics, Information and System Sciences 22, 63-83.
4. Chandrasekharan Rajendran (1992), *“Two Stage flow shop scheduling problem with bicriteria”*, Operational Res. Soc , 43(9), 871-884.
5. Chakarvarthy K. & Rajendrah, C.(1999), *“A heuristic for scheduling in a flow shop with bicriteria of makespan and maximum tardiness minimization”*, Production Planning & Control , 10 (7), 707-714.
6. D. Rebaine, V.A. Strusevich(1998), *“Two-machine open shop scheduling with special transportation times”*, CASSM R&D Paper 15, University of Greenwich, London, UK.

7. Gupta Deepak, Singh T.P et.al(2005) , *“On job block open shop scheduling ,the processing time associated with probability ”*J. Indian Soc. Stat. Oper. Res. Vol.XXVI,No.1-4.pp. 91-96.
8. Gupta, D., Singh, T.P. & Kumar, R.(2007), *“Bicriteria in scheduling under specified rental policy, processing time associated with probabilities including job block concept”*, Proceedings of VIII Annual Conference of Indian Society of Information Theory and Application (ISITA) , 22-28.
9. Gupta, D.& Sharma, S.(2011), *“Minimizing rental cost under specified rental policy in two stage flow shop, the processing time associated with probabilities including break-down interval and job – block criteria”* , European Journal of Business and Management 3(2), 85-103.
10. Gupta D, Sharma S, Seema and Shefali (2011), *“Bicriteria in n × 2 flow shop scheduling under specified rental policy ,processing time and setup time each associated with probabilities including job-block”*, Industrial Engineering Letters, 1(1), 1 -12.
11. Gupta D, Renuka and Singla P(2012), *“A Heuristic Approach For Two Stage Open Shop Scheduling With Transportation Time And Weightage Of jobs Including Job Block Criteria,the Processing Time Associated with Probabilities”*, International journal of scientific and Research publications, ISSN-2250-3153, vol-2, issue 7.
12. Johnson, S.M.(1954), *“Optimal two and three stage production schedule with set up times included”*, Naval Research Logistics Quart. 1(1), pp.61-68.
13. Jackson, J. R. (1956), *“An extension of Johnson’s results on job scheduling”*, Nav. Res. Log. Quar., 3, pp 201-203
14. Maggu P.I. and harbans lal (1989) *“on job block open shop scheduling problem”* PAMS Vol XXIX-pp- 45- 51.
15. Maggu P.L. and Das G., *“Equivalent jobs for job block in job scheduling”*, Opsearch, Vol 14, No.4, (1977), 277-281.
17. Smith, R.D.& Dudek, R.A.(1967) *“A general algorithm for solution of the N-job, M-machine scheduling problem”*, Operations Research 15(1) , 71-82.
18. Yoshida and Hitomi (1979), *“Optimal two stage production scheduling with set up times separated. AIIE Transactions”*, Vol . II, 261-263