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Sampling and Reconstruction of Wireless UWB Pulses by Multi-Stage Exponential Filter

K. Jarvinen[1], S. Ahtiainen[1] , J.T. Olkkonen[2], H. Olkkonen[1]
[1]Department of Applied Physics, University of Eastern Finland, Kuopio, P.O. Box 1627, 70211 Kuopio, Finland. e-mail: hannu.olkkonen@uef.fi
[2]VTT Technical Research Centre of Finland, B.O. Box 1000, 02044 Espoo, Finland

ABSTRACT
In wireless ultra wideband (UWB) technology the sampling and reconstruction of short impulses (Diracs) is an important research object in signal processing society. In this work we introduce a multi-stage exponential filter (MSEF) for sampling and reconstruction of the UWB pulses. The MSEF is constructed from a cascade of filters each having an exponentially descending impulse response. We show that with 2*p*-stage MSEF it is possible to reconstruct UWB pulses consisting of p Diracs from the measurement of at least 2*p* samples. A pole cancellation filter is used to extract the amplitudes and time locations of the Diracs. Robust singular value decomposition (SVD) based subspace method is used to cancel noise interference. The MSEF is applied for sampling and reconstruction of UWB pulses generated by a near range RFID device.

Key words: Impulse train, Dirac distribution, wireless transmission, UWB

1. INTRODUCTION

The information in most of the wireless ultra wideband (UWB) devices is carried out by monocycle Gaussian pulses. However, in year 2002 the FCC restricted the allowed frequency band between 3.1-10.6 GHz for unlicensed UWB transmission [1]. The Gaussian pulse stream does not meet this constraint and other pulse shapes have been introduced to meet the FCC criteria, e.g. the family of the orthogonal UWB pulse waveforms [2,3]. However, the specific pulse generators are relatively difficult to construct. In this work we concentrate on the in low-range wireless ultra wide-band (UWB) communication devices, which transmits pulses consisting of sequential impulses (Diracs). The information is coded to the amplitudes and time locations of the Diracs. Such pulse generators are easy to implement in VLSI [4]. The pulse stream is designed so that its power spectral density coincides with the FCC criteria.

The sampling methods for non band-limited signals (such as impulses and edges) have recently been an interesting research object in signal processing society [5-6]. One of the prominent methods is the sampling scheme with finite rate of innovation (FRI) [7-12]. The key idea in FRI is that the Diracs are fed to an analog circuit, which has a specific impulse response. The output of the sampling filter is measured and the original signal is reconstructed from the discrete samples. Our research group has introduced the parallel sampling scheme, where the signal is fed to the parallel RC circuits, whose outputs are sampled simultaneously [13]. Variants of the parallel sampling scheme include detection of edges and transient waveforms [14-17]. Recently a multichannel (MC) approach was introduced, where the input signal is modulated by a set of sinusoidal waveforms, followed by a bank of integrators [18]. The MC arrangement yields Fourier series coefficients, which enable the reconstruction of the input signal.

In this work we study the FRI-like method aimed at sampling and reconstruction of the UWB pulses. As a sampling device we apply a multi-stage exponential filter (MSEF), whose output is measured sequentially. The reconstruction algorithm is based on the discrete Fourier series representation of the MSEF's impulse response. A novel pole cancellation filter is used to extract the amplitudes and time locations of the Diracs. A robust singular value decomposition method is used to cancel noise interference. We prove that with 2*p*-stage MSEF it is possible to reconstruct UWB pulses consisting of p Diracs from the measurement of at least 2*p* samples. As a practical example we apply the MSEF for the sampling and reconstruction of the UWB pulses yielded by the RFID device.

2. THEORETICAL CONSIDERATIONS

2.1 Sampling of the impulse response

The sequential impulse train $I(t)$ consisting of p Diracs is defined as

$$I(t) = \sum_{i=1}^{p} A_i \, \delta(t - t_i) \qquad (1)$$

where A_i are the amplitudes and t_i the time locations. The impulse train is fed to the multi-stage exponential filter (MSEF) consisting N RC filters in series (Figure 1).
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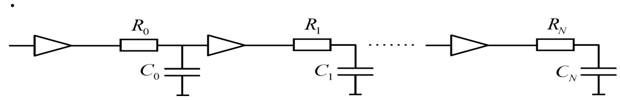

Figure 1: Construction of the MSEF from the exponential filters in series, which are built using unity amplifiers and RC-circuits.

The impulse response of the MSEF is represented by the discrete Fourier series

$$h(t)=\sum_{k=0}^{N-1} a_k e^{j\omega_k t} \quad (2)$$

where the angular frequency $\omega_k = 2\pi k/N,\ \ k = 0,...,N-1$. By denoting $t = nT$, $n = 0,..., N-1$, where T is a sampling interval, the a_k coefficients are computed by the DFT algorithm

$$a_k=\frac{1}{N}\sum_{n=0}^{N-1} h_n\, e^{-2\pi jkn/N} \quad (3)$$

where h_n, $n = 0,..., N-1$ is the discrete-time impulse response of the MSEF.

The output signal of the MSEF is yielded by the convolution

$$x(t)=I(t)*h(t)=\int_{-\infty}^{\infty} I(\tau)h(t-\tau)\,d\tau. \quad (4)$$

By inserting (1) and (2)

$$\begin{aligned} x(t)&=\int_{-\infty}^{\infty}\sum_{i=1}^{p} A_i\,\delta(\tau-t_i)\sum_{k=0}^{N-1} a_k e^{j\omega_k t-\tau}\,d\tau \\ &=\sum_{i=1}^{p} A_i \sum_{k=0}^{N-1} a_k e^{j\omega_k (t-t_i)}. \end{aligned} \quad (5)$$

Changing the order of the summations we have

$$x(t)=\sum_{k=0}^{N-1} a_k e^{j\omega_k t}\sum_{i=1}^{p} A_i e^{-j\omega_k t_i}. \quad (6)$$

By denoting

$$b_k=\sum_{i=1}^{p} A_i e^{-j\omega_k t_i} \quad (7)$$

we obtain

$$x(t)=\sum_{k=0}^{N-1} a_k b_k e^{j\omega_k t}. \quad (8)$$

The b_k coefficients can be now computed by the DFT algorithm as

$$b_k=\frac{1}{a_k N}\sum_{n=0}^{N-1} x_n\, e^{-2\pi jnk/N} \quad (9)$$

where x_n is the sampled output signal of the MSEF. We may write (7) as

$$b_k=\sum_{i=1}^{p} A_i\, e^{-j(2\pi k/N)t_i} \quad (10)$$

and by denoting

$$\lambda_i = e^{-j(2\pi/N)t_i} \quad (11)$$

we finally obtain

$$b_k=\sum_{i=1}^{p} A_i\,\lambda_i^k. \quad (12)$$

2.2 Reconstruction of the amplitudes and time locations of the Diracs

The z transform of the b_k sequence is

$$\begin{aligned} Z\{b_k\} &= \sum_{k=0}^{\infty}\sum_{i=1}^{p} A_i\,\lambda_i^k z^{-k} = \sum_{i=1}^{p} A_i \sum_{k=0}^{\infty}\lambda_i^k z^{-k} \\ &= \sum_{i=1}^{p}\frac{A_i}{1-\lambda_i\, z^{-1}} \end{aligned} \quad (13)$$

Let us define the pole cancellation filter (PCF) as

$$H_{pc}(z)=1+\sum_{n=1}^{p} h_n\, z^{-n}=\prod_{j=1}^{p}(1-\lambda_i\, z^{-1}) \quad (14)$$

and the product filter $P(z)$ as

$$\begin{aligned} P(z) &= Z\{b_k\}H_{pc}(z)=\sum_{i=1}^{p}\frac{A_i}{1-\lambda_i\, z^{-1}}\prod_{j=1}^{p}(1-\lambda_j\, z^{-1}) \\ &= \sum_{i=1}^{p} A_i \prod_{\substack{j=1 \\ j\neq i}}^{p}(1-\lambda_j\, z^{-1}). \end{aligned} \quad (15)$$

We may observe that the roots of the PCF equal the roots of the $P(z)$. The impulse response of the product filter is

$$p_n=\sum_{k=0}^{p} b_{n-k}\, h_k \quad (16)$$

For solution of the roots of the PCF we set $p_n = 0$ for $n \geq 0$. This yields the matrix/vector equation

$$\begin{bmatrix} b_{2p-1} \\ b_{2p-2} \\ \vdots \\ b_p \end{bmatrix} + \begin{bmatrix} b_{2p-2} & b_{2p-3} & \cdots & b_{p-1} \\ b_{2p-3} & b_{2p-4} & \cdots & b_{p-2} \\ \vdots & \vdots & \vdots & \vdots \\ b_{p-1} & b_{p-2} & \cdots & b_0 \end{bmatrix}\begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_p \end{bmatrix} = \mathbf{0} \quad (17)$$

$$\Leftrightarrow \mathbf{b}+\mathbf{B}\,\mathbf{h}=\mathbf{0} \Rightarrow \mathbf{h}=\mathbf{-B^{-1}b}$$

The solution of the p coefficients of the pole cancellation filter requires the knowledge of the $2p$ values of the b_k sequence. This needs the application of the $2p$-stage MSEF. The polynomial $[1\ h_1\ h_2 \ldots h_p]$ has the roots $z_i = e^{-j(2\pi/N)t_i}$ $(i = 1, 2,..., p)$, which gives the lime locations as $t_i = jN \log z_i / (2\pi)$. The amplitudes A_i $(i = 1, 2,..., p)$ can be solved from (12) by writing the matrix/vector equation

$$\begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_{p-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_p \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_1^{p-1} & \lambda_2^{p-1} & \cdots & \lambda_p^{p-1} \end{bmatrix}\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_p \end{bmatrix} \quad (18)$$

$$\Leftrightarrow \mathbf{b}=\mathbf{L}\,\mathbf{a} \Rightarrow \mathbf{a}=\mathbf{L^{-1}}\,\mathbf{b}$$

To summarize, the reconstruction of p impulses requires the knowledge of the b_k sequence. This requires the measurement of at least $2p$ samples from the $2p$-stage MSEF output.

2.3 Noise cancellation

Some degree of noise is generated in electronic circuits, which interferes the results. The solution of the b_k coefficients from (9) requires noise cancellation of the data vector $x = [x_0\ x_1 \cdots x_M]^T$. In the presence of noise we have used the singular value decomposition (SVD) based subspace method for reducing the noise in measurement signal. Let us construct the Hankel matrix containing the measurement values $x_n = x(nT)$

$$H = \begin{bmatrix} x_0 & x_1 & \cdots & x_{M/2} \\ x_1 & x_2 & \cdots & x_{M/2+1} \\ \vdots & \vdots & \vdots & \vdots \\ x_{M/2} & x_{M/2+1} & \cdots & x_M \end{bmatrix} \tag{19}$$

where the antidiagonal elements are identical. To obtain a full matrix (16) M must be even. The SVD of the matrix H is

$$H = U \Sigma V^T \tag{20}$$

where U and V are unitary matrices. Σ is a diagonal matrix consisting of the singular values in descending order. The decomposition (17) can be separated as

$$H = \begin{bmatrix} U_s & U_n \end{bmatrix} \begin{bmatrix} \Sigma_s & 0 \\ 0 & \Sigma_n \end{bmatrix} \begin{bmatrix} V_s & V_n \end{bmatrix}^T \tag{21}$$

$$= U_s \Sigma_s V_s^T + U_n \Sigma_n V_n^T = H_s + H_n$$

where Σ_n contains the smallest singular values. The H_n matrix can be considered to belong to the noise subspace [19]. The matrix H_s is related to the noise free signal subspace. The dimension of the signal subspace equals the number of stages in the MSEF circuit. The constructed signal matrix H_s is not precisely Hankel matrix, but some variation occurs in the antidiagonal elements. We reconstructed the noise free Hankel matrix by replacing the antidiagonal elements by their mean values. This enables the computation of the noise cancelled $x_n (n = 0,1,2,...,M)$ sequence.

3. EXPERIMENTAL RESULTS

The theoretical results were warranted by extensive numerical simulations. The number of the impulses in one burst varied between 1- 3. The number of RC-filters in MSEF was in the range 3 - 7. The amplitudes of the impulses were randomly distributed between limits 0.2 - 1.0. The simulations proved the essential constraint that for the recovery of p impulses at least 2p samples must be measured in the 2p-stage MSEF output. In every case the present method recovered the amplitudes and time locations with a machine precision.

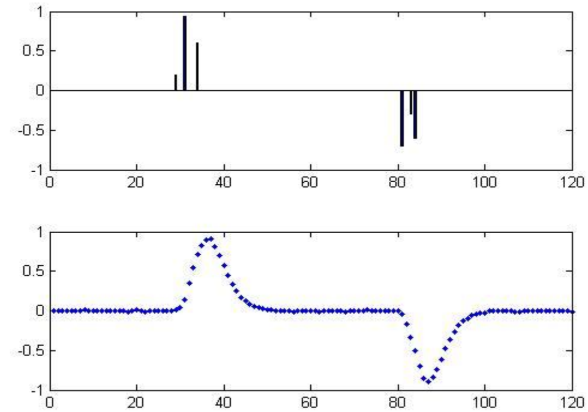

Figure 2: The output signal of the MSEF comprising of six RC circuits measured with a high-speed memory oscilloscope (sampling rate 1 GHz). Each UWB pulse consisted of three Diracs.

In a prototype MSEF the RC circuits were separated via the unity gain buffer amplifiers. A careful electrical shielding and a large area grounding plate were used in the construction of the MSEF circuit. The UWB pulses yielded by a programmable impulse generator were fed to the wireless UWB transmitter, which was a part of the commercial RFID device. Each UWB pulse contained three Diracs. The pulse train was measured at a distance of three meters using the UWB antenna, which was fed to the MSEF. The output of the MSEF was measured with a high speed memory oscilloscope (sampling rate 1 GHz). A typical measurement is described in Figure 2.

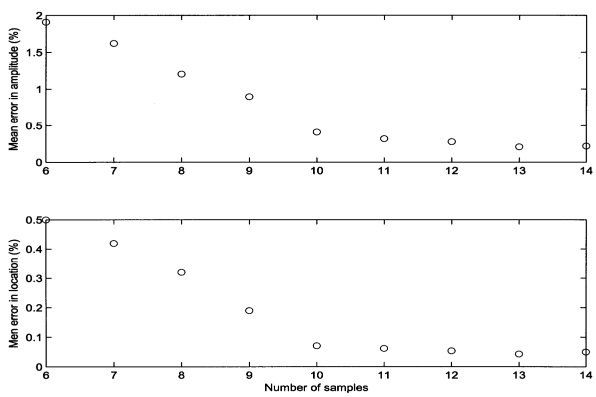

Figure 3: The mean percentage errors in the amplitudes and time locations versus the number of samples used for the reconstruction algorithm.

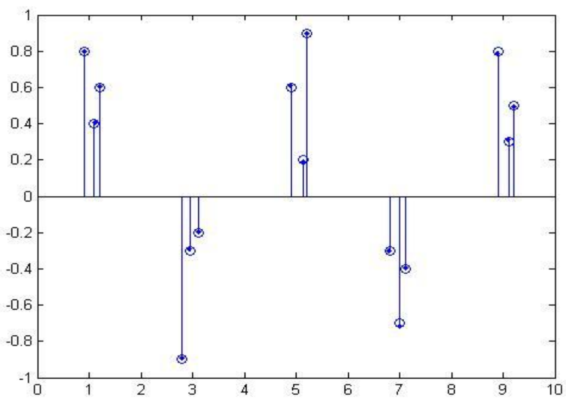

Figure 4: The reconstruction of the three sequential UWB pulses each containing three Diracs. The open circles denote the original impulses and dots the reconstructed amplitudes and time locations. The time scale is in 10 nsec. The MSEF consisted of six serial RC circuits. Each pulse containing three Diracs was reconstructed from the 14 measurement points

The prototype MSEF recovered the amplitudes and time locations of the UWB pulses with a good accuracy. The reconstruction performance was deteriorated when the number of stages in the MSEF exceeded the 2p rule for the UWB pulses containing p impulses. In all subsequent tests the number of stages was six to match to the three impulses involved in the UWB pulses. In spite of the SVD based noise cancellation method the reconstruction error appeared to be sensitive to additive noise. By increasing the number of samples the experimental results showed a significantly improved accuracy. The mean error in the amplitudes was 1.9 % and in the locations 0.5 %, when the minimum of six samples was used to reconstruct one UWB pulse (Figure 3). The mean reconstruction error decreased to 0.2 % in the amplitudes and 0.05 % in the locations, when 14 samples were used for the reconstruction. The reconstruction error decreased rapidly in the range 6-10 samples and then with a slower decay in the range 10-14 samples (Figure 3). Fig. 4 gives an example of the reconstruction of the sequence of five UWB pulses.

4. CONCLUSIONS

The present work describes the FRI-like method for sampling and reconstruction of the pulses in the wireless UWB technology. The MSEF network lengthens the UWB pulses for sampling by the analog-to-digital converter. The main reason for the selection of the MSEF is that the network consists of only single pole RC-filters yielding real-valued impulse response. More elaborate network creates complex exponential waveforms and weighting coefficients, which would make the reconstruction algorithm more complicated. The key idea for the reconstruction of the amplitudes and time locations of the sequential Diracs in UWB pulses is the application of the pole cancellation filter (14).

Recently a multichannel (MC) approach was introduced, where the input signal was modulated by a set of sinusoidal waveforms, followed by a bank of integrators [14]. The MC arrangement yields Fourier series coefficients, which enable the reconstruction of the input signal. In the case of two sequential impulses the performance of the MC approach was poorer than the results obtained by the parallel bank of exponential filters [13] in the presence of noise below SNR<50 dB, but significantly higher at SNR>50 dB. However, in the case of ten sequential impulses the performance of the MC approach was significantly higher compared with the parallel bank of exponential filters. On the other hand, the MC arrangement is much more complex compared to the circuit consisting of the parallel exponential filters.

The theoretical formulations of the reconstruction algorithm were warranted by throughout simulations. In practical measurements the SVD based noise cancellation method had to be applied due to noise interference. Because the computational complexity of the SVD algorithm is $O(N^3)$, the real-time applications of the present method are restricted by a relatively low transmission rate. However, since the information is coded to both the amplitude and the time locations of the Diracs, the number of transmitted pulses can be considerably lower compared with the conventional UWB methods. Also the lower number of transmitted pulses reduces the RF radiation load. The present FRI-like method is intended primarily on applications, where a limited number of information is wirelessly transmitted, such as the near range RFID technology.

Our experimental results show that the reconstruction error can be decreased by increasing the number of samples at least twice the minimum (Figure 3). In this work we used 1 GHz sampling rate and if 20 samples are taken per one UWB pulse the repetition rate is maximum 50 MHz. To increase the pulse repetition rate by one order would require the use of the 10 GHz GaAs analog-to-digital converter (ADC). However, the lower cost flash-type CMOS and GeSi ADCs are under development and their speed is rapidly increasing. This would motivate the use of the wireless UWB pulse transmission systems to replace the cables and fibre optic links e.g. in industrial electronics, robotics and medical instrumentation.

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