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An Analytical Reliability Performance Assessment of Cell-Free Massive MIMO Systems for 5G and Beyond Cellular Networks

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ABSTRACT

In today's realm, the 5G cellular networks offer a new era of fully digital environment by connecting people anywhere anytime. This era requires inspiring demands to the networks in terms of spectral and energy efficiencies, low-latency and ultra-reliable communications. A major development to deal with these unparalleled demands is the introduction of Cell-Free Massive MIMO (CF mMIMO) systems to the cellular network. In recent years, mathematical foundations have been laid for a CF mMIMO communication system to evaluate the spectral and energy efficiencies given the number of access points. Various existing literatures have adopted mathematical models to formulate closed-form expressions for these efficiencies using uncorrelated Rayleigh fading for simple linear processing detectors. In this paper, we try to address a compelling and interesting question which has not received much attention lately. The question is how to evaluate the reliability of a connected user terminal in both a CF mMIMO and conventional mMIMO system? We answer this question by first analyzing the signal models for both systems assuming exact channel estimates and consider an uncorrelated Rayleigh fading channel model. The exact Bit Error Rate, as a reliability assessment, in closed-form are then derived based on strong mathematical foundations, which are simple to numerically evaluate in Matlab R2022, for MPSK and MQAM modulation schemes. The mathematical models presented in this work can be used in 5G and Beyond 5G networks to evaluate the distributions of error rates for a user given the number of Access points or Base Station antennas and modulation scheme. Machine Learning algorithms can be applied to the analytical expressions with a view to predict the error performance of a user terminal given the fading characteristic of the propagation environment.

Key words: Cell-free Massive MIMO, wireless communications, Rayleigh fading channel, 5G networks, linear receivers, Error probability integral

1. INTRODUCTION

The fifth-generation (5G) wireless networks have achieved numerous enhancements namely bandwidth expansion, air interface upgrade and network magnification by employing modern Massive MIMO and mmwave physical layer technologies. With the recent introduction of connected intelligence and a wide variety of applications namely industrial Internet of Things (IoT), brain computer interfaces and tactile internet the existing 5G systems require many challenges to cope with these new technologies. In this context, it is crucial to research on cell-free massive MIMO communications, which take the advantages of distributed systems and massive MIMO coupled with solutions to further boost spectral and energy efficiencies of cellular networks. The near future world of wireless cellular systems will comprise of huge amount of smart devices, User Terminals (UTs), data volume traffic and a wide range of wireless applications which will have to communicate with low latency, high data rates as well as low error probability rates so as to meet the objectives of Beyond 5G (B5G) and 6G systems.

Lately, it is reported in [1] that traffic from smart devices will increase by ten times and overall mobile data traffic will rise by eight times. Novel solutions for future wireless systems are dependent on both the expansion in spectrum and cells densification to increase the achievable data rates. The issue with these requirements is that the cost of implementing them is high in terms of hardware. However, one way to boost the cell throughput data rates without increasing cost of deployment is by increasing the spectral and energy efficiencies of the underlying wireless access network.

Furthermore, to meet the requirements of 5G and future cellular systems with regards to low latency, high data throughputs as well as ultra-reliable data communications the Massive MIMO technology is considered. The technology calls for installing a BS with hundreds of antennas that communicate single-antenna UTs [2]. It has been shown in various literatures that Massive MIMO can boost spectral efficiencies of UTs by an order of ten times without bandwidth expansion and additional cost of implementation. Massive MIMO will continue to play a crucial role in supplying high data rates with ultra-reliable low latency characteristics and recently reconfigurable technologies have been introduced [3]. In 5G and future cellular networks, linear receivers are implemented in the BS so that the Massive MIMO system can transmit and receive data as well as pilot signals to and from the user terminals in the uplink channel. These receivers achieve a property of optimality meaning that their performances are similar to those of Maximum Likelihood (ML) receivers.

In a cell-free Massive MIMO communication system no cell and cell boundaries exist thereby yielding high data rates, ultra-low latency, excellent coverage area, high energy efficiency as well as ultra-high reliability communications [4,5]. The basic operating principle of cell-free system is to deploy a large number of distributed Access Points (APs) connected to a central processing unit to serve all user terminals in a wide coverage area. Cell-free networks offer better connectivity as compared to the conventional massive ones as well as array gains [6].

Due to the complex fading phenomena of wireless channels it is crucial to use mathematical models for characterizing the nature of the fading so that services delivered by mobile and wireless networks can be reliably delivered. To this end, performance assessments in terms of reliability or more precisely error probabilities become very important to determine the level of reliability of a communication system. In order to calculate the probability of error for a specific modulation scheme under various fading channel conditions the conditional symbol error probability (conditioned over the fading statistics) is averaged over the probability density function (pdf) of the fading channel amplitude [7]. This computation is possible by using the Gaussian *Q*-function or the complementary error function *erfc()*.

It is well-known that employing cell- free Massive MIMO technology increases the spectral as well as energy efficiencies of user terminals in the cellular network. Numerous mathematical formulations have been provided in existing literatures to justify the boost in efficiencies. The error rate computations of linear receivers for Massive MIMO systems have not been fully studied in recent years although ultra-reliable communication is set as a primary goal of present and future cellular systems. In this context we try to determine the error rate distributions of practical receivers in the conventional and cell- free Massive MIMO regime.

The remainder of this manuscript is organized as follows. Section 2 relates the recent studies to our research work and a proposed system model is described in Section 3. The signal analysis for an uplink channel of the conventional and cell-free Massive MIMO system under perfect CSI channel conditions is elaborated in Section 4. Exact closed-form BER expressions of linear receivers are derived in Section 5 and analytical results are presented, compared and discussed in section 6. Finally, section 7 concludes our work.

2. RELATED WORKS

Recently the performance of conventional multi-user massive MIMO system with error vector signals for 5G networks has been analytically presented in terms of spectral and energy efficiencies [8]. Moreover, the error probability of Massive MIMO systems with Orthogonal Frequency Division Multiplexing (OFDM) and ZF receiver has been recently evaluated in closed-form in [9]. It is shown that the error probability accuracy is improved by using the Newmann series expansion and the effective noise pdf. A cloud-based cell-free massive MIMO system for B5G/6G networks has been studied in terms of symbol error rates with near optimal detection algorithms [10]. Non-identical large-scale fading coefficients and channel estimation errors under imperfect CSI have been considered. It is shown that the proposed detectors outperform conventional MRC and MMSE counterparts.

The application of linear receivers such as MRC, ZF and MMSE to successive interference cancellation methods for the uplink of a Massive MIMO system has been considered in [11]. In the high SNR regime the outage performance of these receivers are studied and it is shown that when compared to classical linear receivers these modified ones yield higher diversity and coding gains which grows as the number of user terminals increase. In [12] the error performance of an uplink multi-cell multi-user Massive MIMO system with spatial modulation is studied over composite Rayleigh fading channels. The ZF receiver is considered and imperfect CSI is assumed at the base station. With pilot contamination the BER is derived and the asymptotic error performance is evaluated. Simulation results verified the validity of all theoretical analysis.

The outage probability and achievable rate of Massive MIMO system with Digital to Analog Converters (DACs) of mixed-level resolution have been derived in [13]. A matched-filter precoding receiver is considered at the base station and the SINR distribution is formulated. Numerical results show that as the number of users increases the system's achievable rate also increases while a decrease in the outage probability is observed.

In [14] the pdf of the channel fading between a base station, an array of large intelligence surfaces (LIS) and a single antenna user is investigated. The exact BER with MQAM and BPSK modulation schemes is derived considering the Nakagami-m channel. It has been shown through Monte Carlo simulations that all derived expressions perfectly match the simulated results. The majority of studies carried out in the Massive MIMO communication system focuses on evaluating the ergodic capacity of both uplink and downlink but the error performance analysis has been omitted [15-21]. Recent studies in evaluation of error floors with regards to practical performance have been reported in [22-25] in terms of pairwise and outage error probabilities. Moreover, these performance assessments have been evaluated based on the Signal to Interference Noise Ratio (SINR) which in turn depends on the fading environment.

Numerous research findings have been presented from a signal processing perspective in the literatures [26-32] for CF mMIMO systems with limited capacity links that connects the APs to a central processing unit. Moreover, extensive research has been carried out recently on analyzing various digital beamforming techniques that can be applied to cell-free systems with a view to achieve practical deployments. In [33], conjugate beamforming was employed in the receiver of an AP and closed-form expression for the spectral efficiency was derived and in [34] emphasis were laid on the problem of selecting the AP with MRC and ZF receivers. The outage probability analysis of uplink cell-free Massive MIMO system with user mobility has been derived and verified through numerical simulations in [35]. It has been shown that integrating cell-free systems in 6G cellular networks is vital to satisfy spectrum and resource allocations.

3. PROPOSED SYSTEM MODEL

In this work, for the conventional massive MIMO system we propose the uplink channel of a T BS antennas communicating with K single-antenna User Terminals (UTs) simultaneously in a single cellular structure. Assume that the uplink mean transmit power of the k^{th} user is given by p_u and that all UTs transmit their data and pilot symbols in the same time-frequency resource over the Massive MIMO system operating in the Time Division Duplex (TDD) mode. We further employ a fading model that characterizes fast fading, geometric attenuation, log-normal shadow fading and is denoted by β_k.

Building on the prior works of [36] we propose a CF mMIMO system with T single-antenna APs and K single-antenna users who are distributed randomly in a wide area. Moreover, the users are considered to use the same time-frequency resource block employing matched-filter beamforming. Let β_{tk} be the large-scale fading coefficient between the k^{th} user, q_k is the symbol of user k, τ^{cf} is the uplink training period and also ρ_p is the SNR of a pilot signal. If we assume that ρ_u refers to the SNR of a user and the power control coefficient of user k is denoted as η_k then it can be shown from [36] that $\gamma_{tk}= \tau^{cf} \rho_p \beta_{tk}^2$ considering the fact that pilot sequences are pairwisely orthogonal.

The received signal vector, at every symbol time interval, is denoted as r which is of dimension $Tx1$ over the BS and given by (1). The transmitted signal vector,x, is selected from the MPSK or MQAM signal constellation with M order of modulation. H is the channel matrix which consists of uncorrelated Rayleigh fading coefficients that are independent and identically distributed (i.i.d) complex Gaussian random variables modeled as $H\sim CN(0,\sqrt{\beta_k})$. Moreover, the complex AWGN noise vector is represented as n whose entries are i.i.d random variables with distribution $n\sim CN(0,\sigma^2)$ where $\sigma^2 = {N_o}/{2T_s}$, N_o being the one-sided noise power spectral density and T_s indicates the symbol time interval. Assuming perfect CSI, if E_s is the average symbol energy of the k^{th} UT in the cell, then its average energy per bit will be expressed by $E_b=E_s/log_2M$.

$$y = \sqrt{p_u}H\boldsymbol{x} + n \qquad (1)$$

4. SINR ANALYSIS

Assume that a linear receiver with matrix B of dimension MxK is employed at the BS of the Massive MIMO system. Under perfect CSI, the received signal r_k at the BS for k^{th} user can be written as shown by (2) where b_k and h_k are the k^{th} columns of matrices B and H respectively. The SINR of the k^{th} UT is formulated from (2) and given by (3).

$$r_k = \sqrt{p_u}\, b_k^H h_k x_k + \sqrt{p_u} \sum_{i=1, i\neq K}^{K} b_k^H h_i x_i + b_k^H n \qquad (2)$$

Signal *Interference* *noise*

$$SINR_k = \frac{p_u |b_k^H h_k|^2 \, |x_k|^2}{p_u \sum_{i=1,i\neq k}^{K} |b_k^H h_i| |x_k|^2 + \|b_k\|^2} \qquad (3)$$

In this analysis we consider the following:
$|x_k|^2 = \frac{E_s}{T_s}$ and $\sigma^2 = {N_o}/{2T_s}$ as well as $E_b=E_s/log_2M$.

For the cell-free system we consider that all K users in the uplink send their data simultaneously to the APs and prior to the transmission phase the k^{th} user weighs its data symbol q_k by the power control coefficient $\sqrt{\eta_k}$ where $0< \eta_k<1$. Hence the received signal at the t^{th} AP is given as follows.

$$r_k = \sqrt{\rho_u} \sum_{i=1}^{K} \sqrt{\eta_k} h_i q_i + n \qquad (4)$$

4.1 MRC receiver

The MRC receiver assumes that both the channel and precoder matrix coefficients are equivalent meaning that the vector element $b_k = h_k$ which when replaced in (3) will give (5) and after some simplifications we can obtain the SINR of the MRC receiver which is given by (6). In our analysis we consider the following parameters: $|x_k|^2 = {E_s}/{T_s}, \sigma^2 = {N_o}/{2T_s}$, $E_b = \frac{E_s}{log_2 M}$.

$$SINR_k^{MRC} = \frac{p_u \|h_k\|^4 \, |x_k|^2}{p_u \left(\sum_{i=1,i\neq k}^{K} \beta_i\right) \|h_k\|^2 |x_k|^2 (K-1) + \|h_k\|^2 \sigma^2} \qquad (5)$$

$$SINR_k^{MRC} = \frac{p_u \|h_k\|^2}{p_u \left(\sum_{i=1,i\neq k}^{K} \beta_i\right)(K-1) + \frac{\sigma^2}{|x_k|^2}} \qquad (6)$$

We consider that K UTs arrive with the same transmit power at the BS together under prefect CSI and that the spectral norm of the channel matrix follows a chi-square distribution with $2T$ degrees of freedom [37]. Hence by replacing $\|h_k\|^2$ with $2T$ and other parameters given earlier we arrive at the closed-form expression of (7) for the SINR of a MRC receiver.

$$SINR_k^{MRC} = \frac{2T p_u}{p_u \left(\sum_{i=1,i\neq k}^{K} \beta_i\right)(K-1) + \frac{1}{2\left({E_b}/{N_o}\right) log_2 M}} \qquad (7)$$

4.2 Zero-Forcing receiver

The application of a Zero-Forcing receiver at the BS implies complete annulation of the interference which results in higher SINR of the user terminals. Therefore, we simplify the signal and SINR models of (2) and (3) to (8) and (9) as shown below. The spectral norm in this case follows a chi-square distribution with $2(T-K+1)$ degrees of freedom and so will the SINR as well [37].

$$r_k = \sqrt{p_u} b_k^H h_k x_k + b_k^H n \qquad (8)$$

$$SINR_k^{ZF} = \frac{p_u |x_k|^2 \, \|h_k\|^2}{\sigma^2}$$

$$SINR_k^{ZF} = 4p_u(T-K+1) \left({E_b}/{N_o}\right) log_2 \qquad (9)$$

4.3 Minimum Mean Square Error receiver

The evaluation of mean SINR of the k^{th} user terminal under prefect CSI is complex because the k^{th} column of the channel and linear receiver matrices give an intractable eigenvalue distribution [38]. However, a conditional eigenvalue distribution can be employed for the MMSE receiver to determine the mean SINR. Therefore, in this work we employ the SINR result of the ZF receiver to be equivalent to that of the MMSE receiver. As elaborated in [38,39] the reason for this similarity in SINR is that the SINR distribution of a MMSE receiver follows the same chi-square distribution to that of a ZF receiver.

4.4 Cell-Free Massive MIMO : Matched –Filter receiver

As elaborated in [36] that all APs are collocated such that $\beta_{tk}=\beta_k$, $\gamma_{tk}=\gamma_k$ and using (4) as well as earlier definitions then the SINR of the k^{th} user in the cell-free system is given by (10).

$$SINR_k^{CF} = \frac{T\rho_u\eta_k\gamma_k}{\rho_u(\sum_{i=1}^{K}\eta_i\beta_i)+\frac{1}{2(E_b/N_o)log_2M}} \quad (10)$$

5. ANALYSIS OF ERROR RATE DISTRIBUTIONS

In what follows we describe in details the evaluations of error rate distributions of linear receivers under perfect CSI condition for the uplink of both conventional and cell-free Massive MIMO communication systems. Several BER expressions which are tractable and easy to numerically evaluate have been reported in [7] and hence in this work we employ them in our approach to determine novel expressions to evaluate error performances in Rayleigh fading channels. The average bit error probability, P_b is found by using (11) where $P(b|\gamma_s)$ refers to the bit error probability conditioned on the instantaneous SINR, γ_s and $f(\gamma_s)$ indicates the probability density function (pdf) of the SINR.

$$P_b = \int_0^\infty P(b|\gamma_s).f(\gamma_s)\,d\gamma_s \quad (11)$$

For the M-ary Phase Shift Keying (MPSK) modulation technique a constant envelope is assumed. Given the desired form of the bit error probability of a MPSK signal over the AWGN channel, to obtain the probability over Rayleigh fading channel we obtain the conditional bit error probability by replacing E_s/N_o with $\gamma_s log_2M$ and then evaluate (11) for the pdf of Rayleigh with the corresponding linear receiver that is, MRC, ZF or MMSE. By applying the principles described above we now proceed with derivation of BERs for our analysis.

$$P_b = \frac{2}{log_2M}\,Q\left(\sqrt{\frac{2E_s}{N_o}}\,sin\left(\frac{\pi}{M}\right)\right) \quad (12)$$

With $E_s/N_o=\gamma_s log_2M$ and $Q(x)=\frac{1}{2}\,erfc(\frac{x}{\sqrt{2}})$ we have the following probability.

$$P(b/\gamma_s) = \frac{1}{log_2M}\,erfc\left(\sqrt{\frac{\gamma_s}{\eta_{MPSK}}}\right) \quad (13)$$

Where $\eta_{MPSK} = \frac{1}{2\,log_2M\,sin^2(\pi/M)}$ and after replacing (13) in (11) we can obtain the BER for MPSK modulation which is given in (14).

$$P_{b,MPSK} = \frac{1}{log_2M}\int_0^\infty erfc\left(\sqrt{\frac{\gamma_s}{\eta_{MPSK}}}\right) f(\gamma_s)\,d\gamma_s \quad (14)$$

The desired form of the bit error probability of M-ary Quadrature Amplitude Modulation (MQAM) signals is given by (15). Using the same principle of derivation as for the case of MPSK signals we derive the BER for MQAM signals given as follows.

$$P_b = \frac{4}{log_2M}\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)Q\left(\sqrt{\frac{3E_s/N_o}{(M-1)}}\right) \quad (15)$$

$$P(b/\gamma_s) = \frac{4}{log_2M}\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right).\frac{1}{2}\,erfc\left(\sqrt{\frac{3\gamma_s\,log_2M}{2(M-1)}}\right) \quad (16)$$

Replacing (16) in (11) and after some simplifications we obtain (17) as follows.

$$P_{b,MQAM} = \frac{2}{log_2M}\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)\int_0^\infty erfc\left(\sqrt{\frac{\gamma_s}{\eta_{MQAM}}}\right) f(\gamma_s)\,d\gamma_s \quad (17)$$

Where $\eta_{MQAM} = \frac{2(M-1)}{3log_2M}$.

5.1 Error Rate distributions for MRC receiver

For Rayleigh fading environments the pdf of the SINR that is, $f(\gamma_s)$ of the MRC receiver has been evaluated in [39] and is given below. Therefore, the integral of (14) can be found using (18) and integration by parts. With some manipulations we can show that the BER of a MRC receiver employing MPSK modulation is given by (19).

$$f^{MRC}(\gamma_s) = \frac{1}{(T-1)!\,(E_b/N_o)^T}\,\gamma_s^{(T-1)}\,exp\left(\frac{-\gamma_s}{E_b/N_o}\right) \quad (18)$$

$$P_{b,MPSK} = \frac{1}{log_2M}\,p^T\sum_{i=0}^{T-1}\binom{T-1+i}{i}(1-p)^i \quad (19)$$

Where $p=\frac{1}{2}\left(1-\sqrt{\frac{\gamma_s}{T\eta_{MPSK}+\gamma_s}}\right)$ and γ_s is given by the SINR of the MRC receiver which has been derived and given by (7). Similarly, the BER of MRC receiver employing MQAM technique can be found by replacing (18) in (17) and evaluating the integral which will give (20) as shown below.

$$P_{b,MQAM} = \frac{2}{log_2M}\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)p^T\sum_{i=0}^{T-1}\binom{T-1+i}{i}(1-p)^i \quad (20)$$

Where $p=\frac{1}{2}\left(1-\sqrt{\frac{\gamma_s}{T\eta_{MQAM}+\gamma_s}}\right)$ and γ_s is given by the SINR of the MRC receiver which has been derived and given by (7). It can be clearly observed from (19) and (20) that we obtained exact closed-form expressions which have not been previously reported for evaluation of the BER of a MRC receiver for a Massive MIMO system under perfect CSI.

5.2 Error Rate distributions for ZF receiver

The ZF receiver follows a chi-square distribution of *2(T-K+1)* degrees of freedom which results in an amendment in the SINR pdf shown in (18). Therefore, the solution of the integrals shown by (14) and (17), using the new SINR pdf of ZF receiver given by (21) as well as the derived SINR of (9), is given by (22) and (23) for MPSK and MQAM respectively.

$$f^{ZF}(\gamma_s) = \frac{1}{(T-K)!\,(E_b/N_o)^{(T-K+1)}}\,\gamma_s{}^{(T-K)}\,exp\left(\frac{-\gamma_s}{E_b/N_o}\right) \quad (21)$$

$$P_{b,MPSK} = \frac{1}{log_2 M}\,p^{(T-K+1)}\sum_{i=0}^{T-K}\binom{T-K+i}{i}(1-p)^i \quad (22)$$

Where $p = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_s}{(T-K+1)\eta_{MPSK}+\gamma_s}}\right)$ and γ_s is given by the SINR of the ZF receiver which has been derived and given by (9).

$$P_{b,MQAM} = \frac{2}{log_2 M}\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)p^{(T-K+1)}\sum_{i=0}^{T-K}\binom{T-K+i}{i}(1-p)^i \quad (23)$$

Where $p = \frac{1}{2}\left(1 - \sqrt{\frac{\gamma_s}{(T-K+1)\eta_{MQAM}+\gamma_s}}\right)$ and γ_s is given by the SINR of the ZF receiver which has been derived and given by (9).

5.3 Error rate distributions for MMSE receiver

The evaluation of the error rates of (14) and (17) for a MMSE receiver with Rayleigh fading interference is complex [37]. However, the integrals can be solved using a reliability approach which has been elaborated in details by Gao et al. [38]. In what follows we describe how to apply this reliability approach to our analysis with a view to find an exact closed-form expression for the BER of a MMSE receiver for Massive MIMO systems.

The channel reliability function $R(\gamma_s)$ for the SINR of a MMSE receiver has been given in [40, eq. 34] and is modified to (24) shown below for a Massive MIMO system for large antenna arrays. Moreover, to find the average BER we apply integration by parts to (14) and (17) which give the reliability function of (25). Leibniz differentiation rule is applied to (25) as well as after some manipulations will give the average BER shown in (26).

$$R(\gamma_s) = \left[\sum_{i=T-K+2}^{T}\frac{\left(\frac{\gamma_s}{\gamma_c}\right)^{i-1}}{\Gamma(i)}\sum_{j=0}^{T-i}\binom{K-1}{j}\frac{\gamma_s^j}{(1+\gamma_s)^{(K-1)}}\right]exp\left(-\frac{\gamma_s}{\gamma_c}\right) \quad (24)$$

$$P_b = P(b/\gamma_s)|_{\gamma_s=0} + \int_0^{\infty}\frac{dP(b/\gamma_s)}{d\gamma_s}\,R(\gamma_s)\,d\gamma_s \quad (25)$$

$$P_b = 1 - \frac{1}{2\sqrt{\pi}\xi}\left[\sum_{i=T-K+2}^{T}\frac{\gamma_c^{(1-i)}}{\Gamma(i)}\sum_{j=0}^{T-i}\binom{K-1}{j}\,\mathrm{F}(x,y,z)\right] \quad (26)$$

$$\mathrm{F}(x,y,z) = \int_0^{\infty}\gamma_s^{(x-1)}(1+\gamma_s)^{-y}.\exp(-z\gamma_s)\;d\gamma_s \quad (27)$$

Where $= i + j - \frac{1}{2}$, y=*K-1*, $z = \frac{1}{\gamma_c} + \xi^2\ with\ \xi = \sqrt{log_2 M}\,sin\left(\frac{\pi}{M}\right)$ for *MPSK* signals and $z = \frac{1}{\eta_{MQAM}} + \frac{1}{\gamma_c} + \xi^2\ with\ \xi = \sqrt{\frac{3\,log_2 M}{2(M-1)}}$ for $MQAM$ signals. γ_s is given by (9).

The evaluation of the integral given by (27) can be performed by employing Table of integrals, series and products from [41, eqs. 9.210-9.211]. Here the result of this integral takes the form of a confluent hypergeometric function written as ${}_1F_1(x;y;z)$. Hence the intergral is performed and after some simplifications we obtained the solution given by (28) below.

$$\mathrm{F}(x,y,z) = \Gamma(x)\left[\frac{\Gamma(y-x)}{\Gamma(y)}\,1F1(x;x-y+1;z) + z^{(y-x)}\frac{\Gamma(x-y)}{\Gamma(x)}\,1F1(y;y-x+1;z)\right] \quad (28)$$

5.4 Error rate distributions for Cell-Free Massive MIMO system

The cell-free massive MIMO system is equipped with a matched filter receiver and in this case the same derivations for the MRC receiver are employed. Hence error rate distributions determined by (22) and (23) are used but now the SINR will be given by (10).

6. ANALYTICAL RESULTS AND DISCUSSIONS

In what follows we report analytical results for all the derived closed-form expressions. With a view to have a practical scenario we consider a hexagonal cellular network structure. The radius (from center to vertex) is taken as 1000 metres which in practice is desirable for the 5G network. It is assumed that no user terminal is closer to the BS than a distance denoted as r_h=100 metres and the large-scale fading is modeled as $\beta_k = \frac{z_k}{(r_k/r_h)^v}$ where z_k refers to the log-normal random variable with standard deviation σ_{shadow}, r_k is the distance between the k^{th} UT and the BS, *v* refers to the path loss. In this case se set $\sigma_{shadow} = 8\ dB$ and *v=3.8.*

Analytical results for MRC receiver are obtained and plotted on Figures 1 and 2 by simulating the equations (19) and (20) for various modulation schemes and SNR per bit. From the results shown we can easily observe that with increasing number of BS antennas the error performance also increases. The findings of BER evaluation carried out by [37] are also plotted in Figure 2 and when compared with the results in this work we can clearly deduce that our findings are far more accurate and better than those achieved in [37]. It can be seen from Figure 2 that with only 200 BS antennas and *K*=16 UTs a BER of 10^{-14} is achievable at low a low E_b/N_o of -15 dB with 4QAM modulation. Moreover, it can be seen from both illustrations that, as expected, with increasing order of modulation schemes the error rates also increase indicating degradation in the performance of the Massive MIMO communication system.

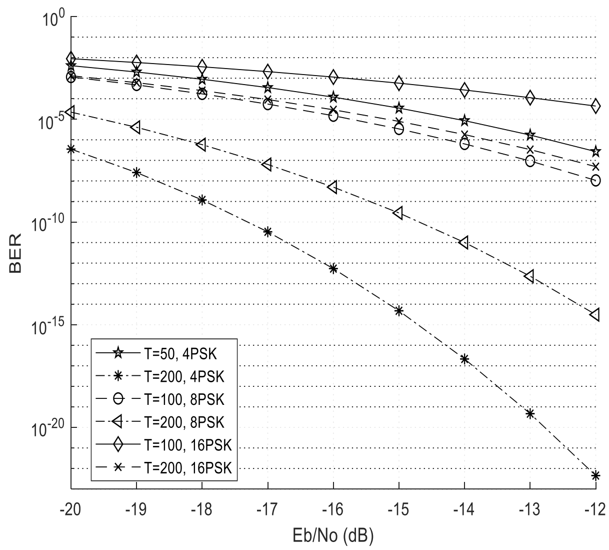

Figure 1: Error performance curves for MRC receiver with MPSK modulation

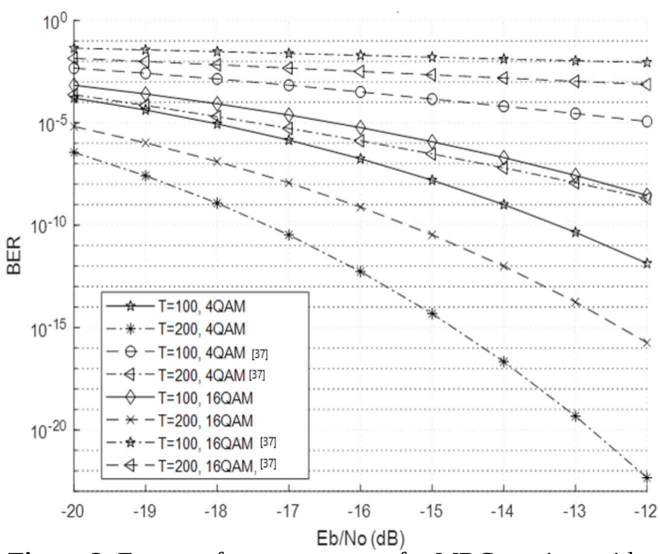

Figure 2: Error performance curves for MRC receiver with MQAM modulation

The distributions of spectral efficiencies with number of BS antennas and MPSK as well as MQAM modulation techniques for MRC receiver are illustrated in Figures 3 and 4. As expected, it can be easily seen that with increasing Massive MIMO antennas the spectral efficiencies also increase. Moreover, we observe that with increasing order of modulation schemes and BS antennas the spectral efficiencies also increases indicating that higher throughput is obtained for the user in the cellular network.

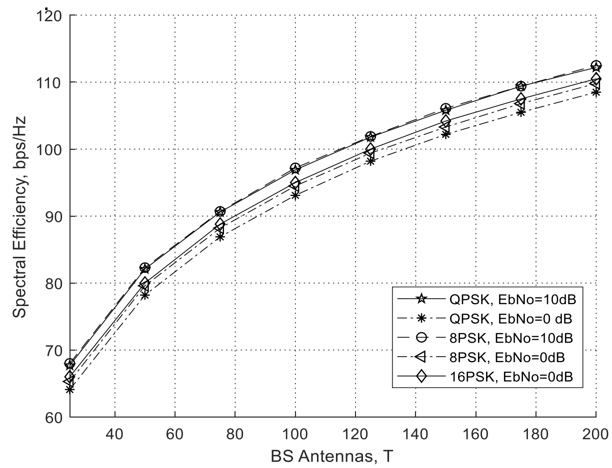

Figure 3: Spectral efficiencies v/s BS antennas for MRC receiver with MPSK modulation

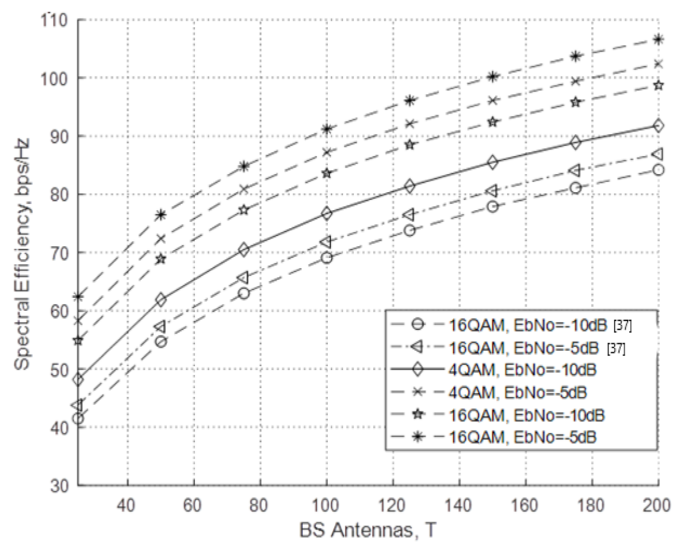

Figure 4: Spectral efficiencies v/s BS antennas for MRC receiver with MQAM modulation

With the application of a ZF receiver in the uplink it can be easily deduced from Figures 5 and 6 that significant error performances are achieved when employing large BS antenna arrays. Furthermore, the results of error rates obtained from [37] are plotted in Figure 6 with a view to compare with our BER analysis. It can be observed that the error performances in this work yield much better results than those obtained in [37]. With 200 BS antennas and QPSK modulation a BER of 10^{-15} can be achieved at an E_bN_o of -13 dB. Moreover, with the same amount of antennas and 4QAM modulation a BER of 10^{-7} can be achieved at an E_bN_o of -18 dB for this work but an increase in E_bN_o to -14 dB is required to achieve the same BER level for the work performed in [37].

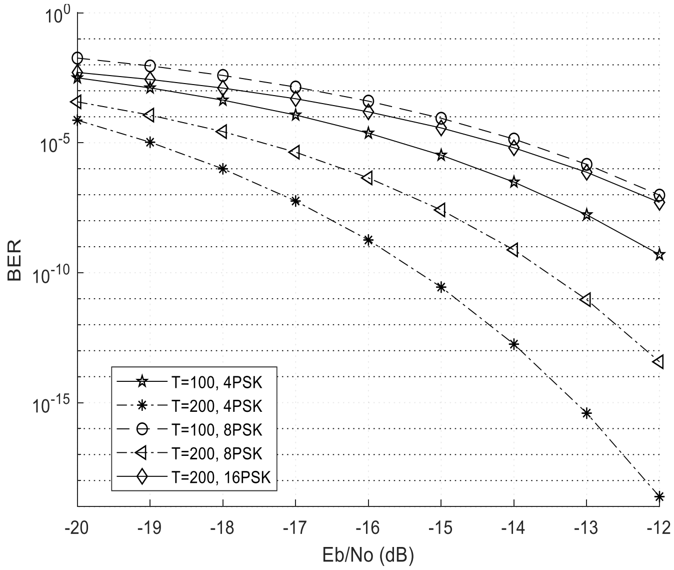

Figure 5: Error performance results for ZF receiver with MPSK modulation

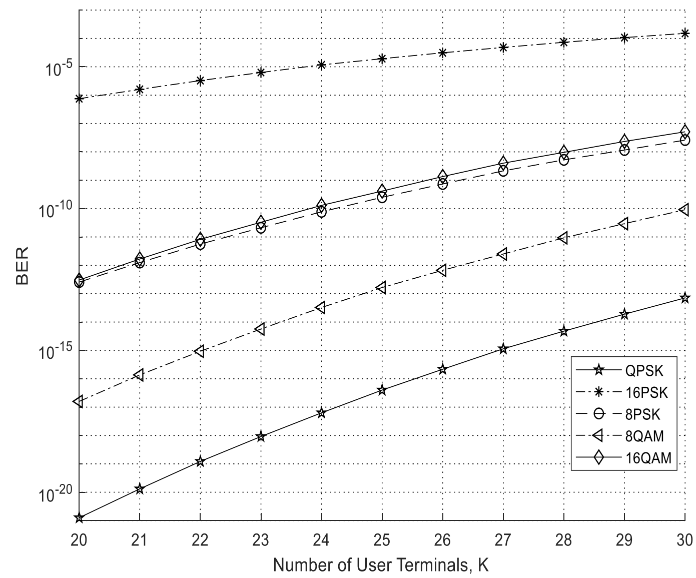

Figure 7: Error rate distribution v/s Number of UTs for MRC receiver (T=100, E_bN_o=-5dB)

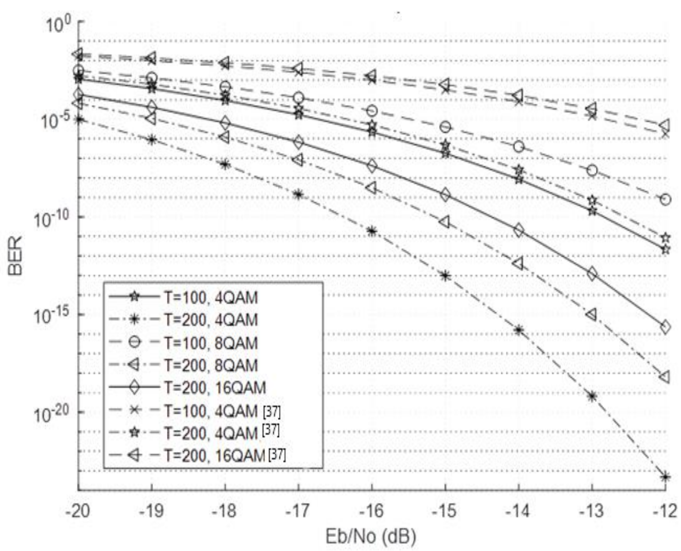

Figure 6: Error performance results for ZF receiver with MQAM modulation

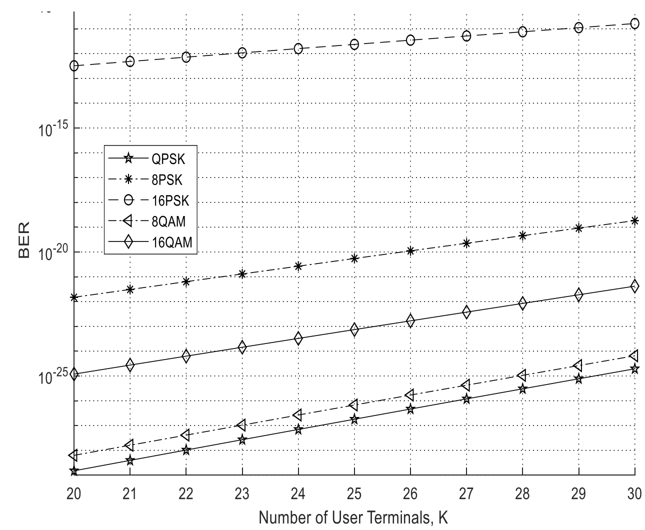

Figure 8: Error rate distribution v/s Number of UTs for ZF receiver (T=100, E_bN_o=-5dB)

The error distribution curves for MRC and ZF curves increasing number of UTs are shown in Figures 7 and 8 respectively. It is interesting to observe that as the number of UTs joining the cell increases the error performance of the user decreases regardless of the modulation scheme and number of BS antennas. This phenomenon can be explained by the fact that the SINR decreases with increasing number of UTs in the cell resulting in significant reduction of the spectral efficiencies of the UTs. Figure 8 indicates that a BER of 10^{-22} can be achieved with 20 users but a reduced BER of 10^{-19} is obtained with 30 users assuming the number of BS antennas is 100 and E_bN_o is -5dB with 8PSK modulation.

Figures 9 and 10 show that for the MMSE receiver high spectral efficiencies as well as low error rates can be achieved. Performance results show that in order to achieve a BER of 10^{-15} only 150 BS antennas and 16 UTs in a single cell with an E_b/N_o of -13 dB is required. As expected, the MMSE receiver is seen to give better error performance than MRC and ZF counterparts.

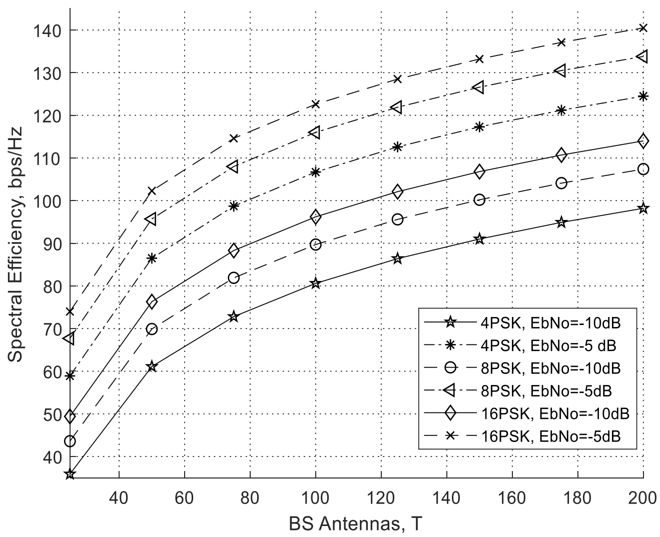

Figure 9: Spectral Efficiencies v/s BS antennas for MMSE receiver with MPSK modulation

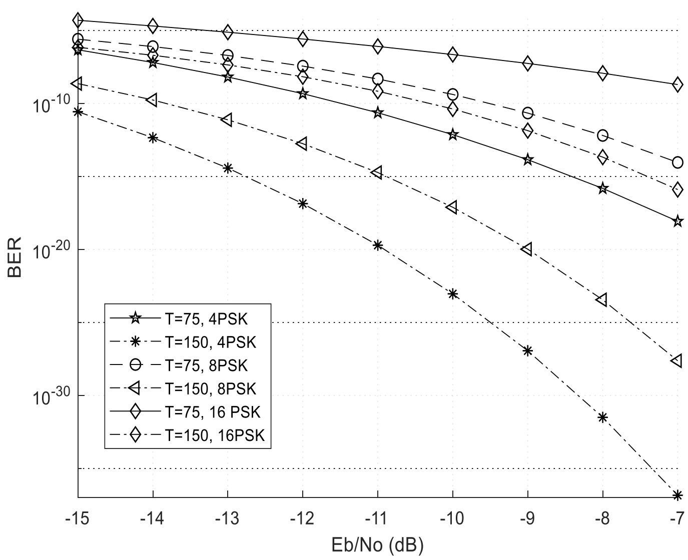

Figure 10: Error performance curves for MMSE receiver with MPSK modulation

The error performances of CF mMIMO systems employing different MPSK modulation schemes and power control coefficients are shown in Figure 11. It can be seen that with increasing number of APs the user error rates diminish significantly and with a low power control coefficient of 0.1 as well as 100 APs the user terminal can achieve a low error rate of 10^{-24}. Figure 12 shows the error rate distributions with varying power control coefficients for different MPSK modulation of a cell-free system equipped with 50 and 100 APs.

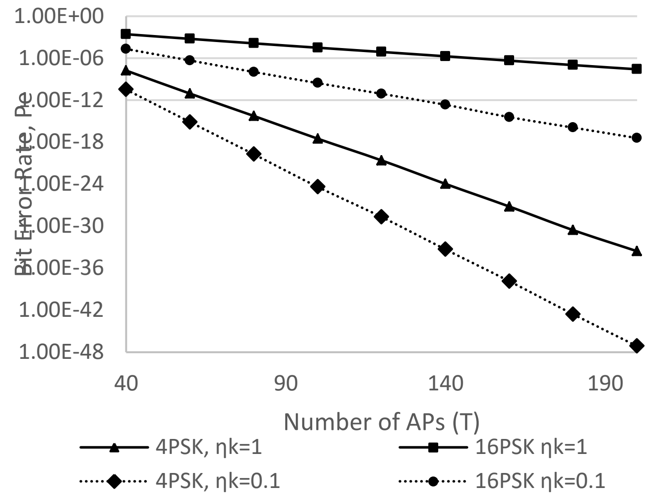

Figure 11: Error performance curves for Cell-Free Massive MIMO systems

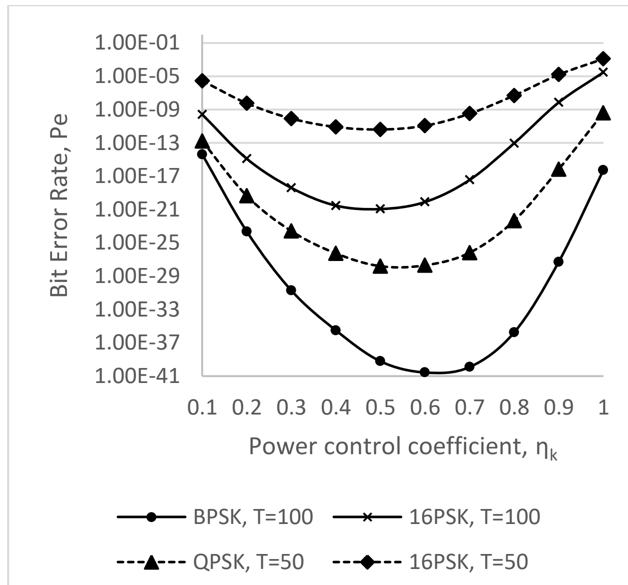

Figure 12: Error rate distributions v/s power control coefficients for CF mMIMO system

Figure 13 compares the error performances between conventional and cell-free massive MIMO systems for both MPSK and MQAM modulation schemes. It can clearly deduce that cell-free systems give better performance than their conventional counterparts.

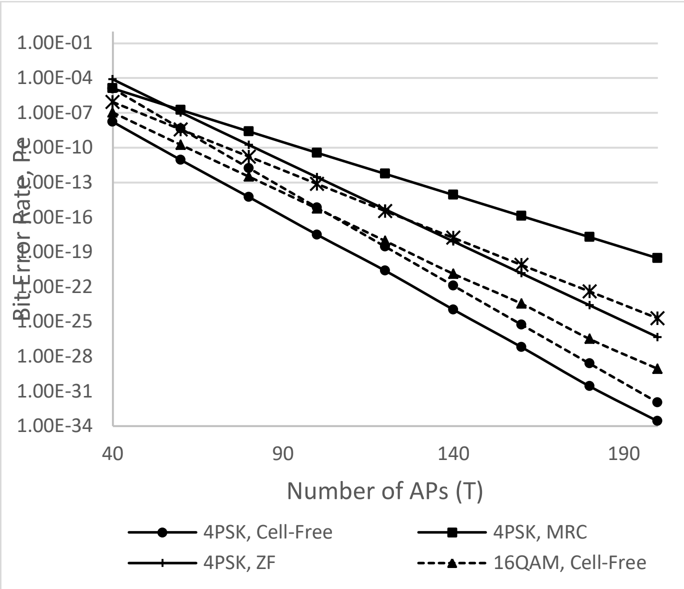

Figure 13: Error performance comparison between conventional and cell-free Massive MIMO

7. CONCLUSION

In this paper, an approach to calculate the error probability distributions of linear receivers for both conventional and cell-free Massive MIMO systems have been presented. The approach is based on evaluating the SINR of the user given the fast-propagating fading environment and then using standard mathematical integrals to compute the error probability distributions for a range of linear receivers.

With the derived closed-form expressions we have shown that this system has the potential to deliver high accuracy and reliability on the uplink of cell-free networks in uncorrelated Rayleigh propagation environments. Through simulation results it is shown that in the limit of an infinite number of BS antennas, for conventional Massive MIMO, and number of APs, for cell-free Massive MIMO, the user error rates significantly reduce resulting in error-free communications. Furthermore, as the number of UTs increases a reduction in error rate is observed.

Quantitatively, performance results for a conventional massive MIMO, employing MMSE receiver, show that in order to achieve a BER of 10^{-15} only 150 BS antennas and 16 UTs in a single cell with an E_b/N_o of -13 dB are required. Furthermore, in a cell-free system simulation results reveal that with 100 APs and power control coefficient of 0.1 the user achieves a low error rate of 10^{-24}. These conclusions are valid under a channel model that includes the effects of small-scale uncorrelated Rayleigh fading. As a future work it is proposed to apply Machine learning algorithms to predict the bit error rate distributions of user terminals by employing the derived expressions.

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