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Analysis of Nonlinear Cylindrical Waveguide Taper Using Modal Matching Technique

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ABSTRACT

Nonlinear tapered waveguide sections are often needed in the microwave/millimeter-wave system to connect the two waveguides of different radii. The presence of such a taper inevitably introduces unwanted parasitic modes and reflections. The modal matching technique is a powerful computer friendly method for analyzing horn antennas or nonlinear waveguides in which the actual profile is replaced by a series of uniform waveguide sections. The waveguides can have any cross section and can be propagating either cylindrical or spherical modes. The mode matching technique involves matching of the total modal field at each junction between uniform sections so that conservation of power is maintained. From this process, the amplitudes of the separate modes at the output of a junction have been deduced in terms of the amplitudes of the mode spectrum at the input to the junction. Each junction along the length has its own scattering matrix. The matrices for all junctions have been be cascaded and an overall scattering matrix has been formed. The overall scattering matrix contains the input reflection coefficient and the output transmission coefficient.

A modal matching field analytical technique has been developed for the analysis of arbitrarily shaped nonlinear waveguide tapers. For a typical practical case, different type, like, linear, parabolic, exponential and raised cosine shape profile have been taken and it have been found that the raised cosine shape profile nonlinear taper is most suitable since it has minimal shape variation at both ends (input as well as output) thereby ensuring minimal reflections, with maximum transmission in the desired mode with appreciably low mode conversion.

The developed analysis should be useful to the microwave practitioners in the analysis and design of nonlinear tapers and for final contour selection having optimum performance for the high power microwave/millimeter wave devices and systems.

Key words : Cylindrical waveguide, field analysis, mode matching technique , nonlinear taper.

1. INTRODUCTION

Inhomogeneous transmission lines and waveguide in form of tapered matching sections or tapered waveguide transition are frequently needed for variety of applications. For the intended use where the output of the high power microwave /millimeter wave devices are connected with the input of the system using these energy, requires a tapered circular waveguide transitions which connectes these two sides having different radii. The problem in the design of these inhomogeneous waveguides is basically of specifying a distributed inhomogeneity for minimum mode conversion and/or minimum reflection over a specific frequency range. If the physical length of the device is at the same time kept at a minimum, the power loss will also be minimum.

The presence of a taper in the waveguide section inevitably introduces unwanted parasitic modes. The requirements for tapers are a good match at the input port and prescribed spurious mode suppression at the output port with a taper length as short as possible. The special difficulties are encountered in the design nonliner tapers when:

1. The taper input the working mode is very close to cutoff. Input reflections as well as forward and backward scattered modes have to be considered.
2. In the case the ratio of the input to output waveguide radii is large so that there is strong excitation of many unwanted modes.
3. For the higher-order operating modes, the higher and lower undesired neighboring modes also gets coupled. TE_{mp} modes with m > 0 couple not only to TE_{mp} modes but also excite TM_{mq} modes.
4. In the case when at the taper throat, the external charge particles are also present, like, in the case of accelerators and microwave/millimeter-wave electron beam devices, which behave like an anisotropic dielectric, excite additional parasitic modes.

The modal matching technique is a powerful method of analyzing horn antennas, will be adopted here for our case of the nonlinear waveguide taper section in which the actual taper profile is replaced by a series of uniform waveguide sections. The waveguides can have any cross section and can be propagating any modes. The mode matching technique involves matching the total modal field at each junction between uniform sections so that conservation of power is maintained. From this process, the amplitudes of the separate modes at the output of a junction can be deduced in terms of the amplitudes of the mode spectrum at the input to the junction. The number of propagating and evanescent modes, which are needed to represent the total power, must be found by trial and error. The power of the modal matching technique stems from the fact that the amplitudes of the modes can be expressed as the components of a scattering matrix. Each junction along the length has its own scattering matrix. The matrices for all junctions can be cascaded and an overall scattering matrix derived from horn. The process of computing the overall scattering matrix can be decoupled from the process of obtaining the elements of a particular scattering matrix. The later will depend on the geometry of the waveguide, but the formal is quite general. Thus, the basic technique can readily be extended to different geometries. The overall scattering matrix for the taper contains the input reflection coefficient and the output transmission coefficient will be computed.

The concept of mode matching at the junction was first attempted in the late 1960s and early 1970s by Wexler [1], Masterman and Clarricoats [2] and English [3]. However, the process of computing the coefficient is lengthy and because of limited computer power was available, it was not possible to do more than simple computations. The computational emphasis was on reducing the number of modes to the minimum so that a numerical solution could be obtained. It was the arrival of powerful computers which enabled the concept to be applied to the analysis of complete tapers or horns. It was firstly developed by James [4], [5] and Kuhn and Hombach [6]. The mode matching technique involves a large amount of computation because there will be large number of modes to be matched across each junction and a large number of junctions along the waveguide/horn [7].

In the present paper, to analyze and design a nonlinear waveguide taper, the modal matching field analysis is presented which involves matching of the total modal field at each junction between uniform sections so that conservation of power is maintained. The overall scattering matrix for the whole taper has been obtained which contains the input reflection coefficient and the output transmission coefficient [7]. The developed technique has been applied for a practical high power gyrotron oscillator under development [8], where the cylindrical output section of the RF interaction cavity operating in the TE_{03} mode is connected with the collector of the device using a nonlinear cylindrical taper. To design this nonlinear taper different shape profile, like, linear, parabolic, exponential and raised cosine is taken and analyzed for the transmission and reflection in the desired operating mode to finalize the optimum shape profile [9].

2. ANALYSIS

The modal matching technique is used for obtaining the overall transmission and reflection coefficient of a waveguide

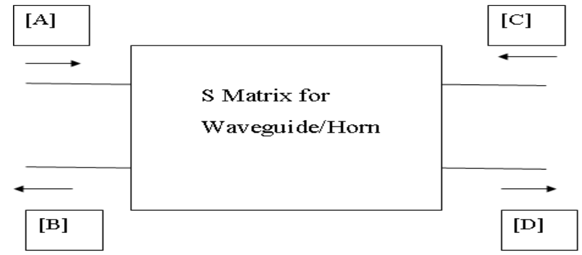

Figure 1: Block diagram showing scattering matrix with forward and reflection coefficients

in the different operating modes. The waveguide is represented as a box, where [A] and [B] are column matrices containing the forward and reflection coefficients of all the modes looking into the waveguide from the source side. Similarly [C] and [D] represent column matrices containing the forward and reflections coefficient of all the modes looking into the aperture of the waveguide from the outside. The characteristics of the tapered waveguide is then given by a scattering matrix[S],

$$\begin{bmatrix} [B] \\ [D] \end{bmatrix} = [S] \begin{bmatrix} [A] \\ [C] \end{bmatrix}, \tag{1}$$

where, [S] is the scattering matrix:

$$[S] = \begin{bmatrix} [S_{11}] [S_{12}] \\ [S_{21}] [S_{22}] \end{bmatrix}. \tag{2}$$

The elements of [S] are the square matrixes describing the power coupling between all the modes at the input with all the modes at the output of the tapered guide. The reflection coefficient for the tapered guide is

$$[B] = [S_{11}][A]. \tag{3}$$

For many tapered guide case, there will only be one mode at the input, in this case [A] will be

$$[A] = \begin{bmatrix} 1 \\ 0 \\ : \\ : \end{bmatrix}. \tag{4}$$

The length of column matrix depends on the number of modes at the output of waveguide. The transmission coefficient of the waveguide, from which the aperture fields are determined as:

$$\left[D\right]=\left[S_{21}\right]\left[A\right] \quad . \tag{5}$$

2.1 Scattering Matrix of a Uniform Section

The scattering matrix elements for a uniform section of guide are

$$\left[S_{11}\right]=\left[S_{22}\right]=\left[0\right] \tag{6}$$

$$\left[S_{12}\right]=\left[S_{21}\right]=\left[V\right] \quad , \tag{7}$$

where, $[V]$ is an $N\times N$ diagonal matrix with the elements V_{nn} = exp($-\gamma_n$), where is the length of the section and γ_n is the propagation constant for the n*th* mode in the waveguide ($1 < n < N$). In principle, the waveguide section could contain lossy material so that γ_n is complex. However, this would lead to extensive computation and is generally unnecessary since the influence of lossy materials can usually be adequately accounted by perturbation approach. The propagation coefficient is normally either purely imaginary ($\gamma_n = j\beta_n$) for travelling modes or is purely real ($\gamma_n = \alpha_n$) for evanescent modes. In both cases, the elements of matrices are real. A substantial number of evanescent modes are included in the analysis. This is because the uniform sections will be relatively short in length so that the amplitude of a decaying wave may still be significant by the time the wave reaches the next junction.

2.2 Scattering Matrix of a Junction

The derivation of the scattering matrix at the junction is more complicated, since it involves matching of total power in all the modes on both sides of the junctions. The number of modes on the left hand side of the junction and the number of modes on the right hand side of the junction can in general be arbitrary. However, it simplifies the analysis and the computational procedure if the number of modes N is the same on both sides of the junction and assumed in the present analysis.

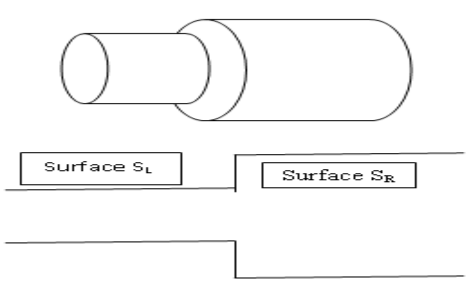

Figure 2: Junction between two sections of cylindrical waveguide

Each uniform waveguide section contains travelling waves in which the transverse electric fields can be represented as a spectrum of N nodes. The transverse electric and magnetic modal functions on the left hand side of the junction are represented by the subscript L (i.e., e_{nL} and h_{nL}), and those on the right hand side of the junction by the subscript R.

The electric and magnetic fields on the left hand side are,

$$E_L = \sum_{n=1}^{N}\left\{A_n \exp\left(-\gamma_n z\right)+B_n \exp\left(\gamma_n z\right)\right\} e_{nL} \tag{8}$$

$$H_L = \sum_{n=1}^{N}\left\{A_n \exp\left(-\gamma_n z\right)-B_n \exp\left(\gamma_n z\right)\right\} h_{nL} \quad , \tag{9}$$

where A_n and B_n are the forward and reflected coefficients of mode n on the left hand side of the junction.

On the right hand side of the junction, the fields have the form

$$E_R = \sum_{n=1}^{N}\left\{C_n \exp\left(-\gamma_n z\right)+D_n \exp\left(\gamma_n z\right)\right\} e_{nR} \tag{10}$$

$$H_R = \sum_{n=1}^{N}\left\{C_n \exp\left(-\gamma_n z\right)-D_n \exp\left(\gamma_n z\right)\right\} h_{nR} \tag{11}$$

where C_n and D_n are the forward and reflected amplitude coefficients of mode n on the right hand side of the junction, looking into the junction.

The total transverse fields must match across the junction. If the junction is at z = 0, then,

$$\sum_{n=1}^{N}\left(A_n + B_n\right)e_{nL} = \sum_{n=1}^{N}\left(C_n + D_n\right)e_{nR} \quad , \tag{12}$$

$$\sum_{n=1}^{N}\left(A_n - B_n\right)h_{nL} = \sum_{n=1}^{N}\left(C_n - D_n\right)h_{nR} \quad . \tag{13}$$

If the cross section area of the waveguide on the left hand side of the junction is S_L and that on the right hand side of the junction is S_R, the boundary conditions give that the transverse electric fields over the area ($S_R - S_L$) will be zero. The fields over the area S_L will be continuous. The continuity of fields and the orthogonality relationship between modes leads to a pair of simultaneous matrix equations.

$$[P]\,[\,[A] + [B]\,] = [Q]\,[\,[C] + [D]\,] \quad , \tag{14}$$

$$[P]^T\,[\,[D] - [C]\,] = [R]\,[\,[A] - [B]\,] \quad , \tag{15}$$

where $[A]$ and $[B]$ are N element column matrices in the section on the left hand side of the junction containing the unknown modal coefficients A_1 to A_N and B_1 to B_N. Similarly,

$[C]$ and $[D]$ are N element column matrices for the right hand side of the junction containing the unknown modal coefficients C_1 to C_N and D_1 to D_N.

The matrix $[P]$ is an $N\times N$ square matrix whose elements are integrals representing the mutual coupled pair between mode i on the left hand side and mode j in the right hand side:

$$P_{ij} = \int_{S_L} \left(e_{iL} \times h_{jR}\right).ds_L \quad . \tag{16}$$

The matrix $[P]^T$ is the transpose of $[P]$, i.e., the rows and columns are interchanged. The matrix $[Q]$ is an $N\times N$ diagonal matrix describing the self coupled power between modes on the right hand side of the junction. The elements are integrals over the area S_R as:

$$Q_{jj} = \int_{S_R} \left(e_{iR} \times h_{jR}\right).ds \quad . \tag{17}$$

Similarly, the matrix $[R]$ is an $N\times N$ diagonal matrix describing the self coupled power between modes on the left hand side of the junction. The elements are integral over the area S_L

$$R_{ii} = \int_{S_L} \left(e_{iL} \times h_{jR}\right).ds \quad . \tag{18}$$

The above three coupling integrals are mode matching equations contain information about the type of waveguide on either side of the junction. They are evaluated for the appropriate taper cross-section. This can be evaluated either analytically or numerically for all modal combinations at each junction. Analytical evaluation is only possible in some cases and involve considerable mathematics though saves considerable computer time.

Equation (16), (17), and (18) are rearranged into the scattering matrix form. This gives the elements of $[S]$ from equations

$$\left.\begin{aligned}
[S_{11}] &= \left[[R] + [P]^T[Q]^{-1}[P]\right]^{-1}\left[[R] - [P]^T[Q]^{-1}[P]\right] \\
[S_{12}] &= 2\left[[R] + [P]^T[Q]^{-1}[P]\right]^{-1}[P]^T \\
[S_{21}] &= 2\left[[Q] + [P][R]^{-1}[P]^T\right]^{-1}[P]^T \\
[S_{22}] &= -\left[[Q] + [P][R]^{-1}[P]^T\right]^{-1}\left[[Q] - [P][R]^{-1}[P]^T\right]
\end{aligned}\right\} \tag{19}$$

In the above analysis, it is assumed that the area S_R is greater than the area S_L. Otherwise, the elements of $[S]$ in equation (2) becomes

$$[S] = \begin{bmatrix} S_{22} \; S_{21} \\ S_{12} \; S_{11} \end{bmatrix} \quad . \tag{20}$$

2.3 Cascading of Scattering Matrix

The whole nonlinear waveguide taper section is divided into a number of uniform sections and junctions. For instance, the whole nonlinear taper has divided into 50 junctions and 49 uniform sections. Each junction or section can be represented by its own scattering matrix with a box as shown in figure 2 and represented by equation (2). $[S]$ represents the scattering matrix of an individual uniform junction or section. The scattering matrix of the total nonlinear taper is made up of a series of scattering matrices as shown in figure 3.

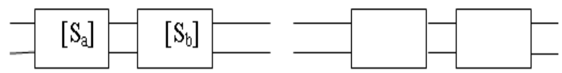

Figure 3: Single Scattering Matrix divided into a number of cascaded S Matrix

Scattering matrices are particularly useful because it is straightforward to cascade two scattering matrix. If these two scattering matrices have elements

$$\left[S^a\right] = \begin{bmatrix} S_{11}^a \; S_{12}^a \\ S_{21}^a \; S_{22}^a \end{bmatrix} \tag{21}$$

and

$$\left[S^b\right] = \begin{bmatrix} S_{11}^b \; S_{12}^b \\ S_{21}^b \; S_{22}^b \end{bmatrix} \quad . \tag{22}$$

Then, the cascaded scattering matrix is

$$\left[S^c\right] = \begin{bmatrix} S_{11}^c \; S_{12}^c \\ S_{21}^c \; S_{22}^c \end{bmatrix} \tag{23}$$

where

$$\left[S_{11}^c\right] = \left[S_{12}^a\right]\left[[I] - \left[S_{11}^b\right]\left[S_{22}^a\right]\right]^{-1}\left[S_{11}^a\right]\left[S_{21}^a\right] + \left[S_{11}^a\right] \tag{24}$$

$$\left[S_{12}^c\right] = \left[S_{12}^a\right]\left[[I] - \left[S_{11}^b\right]\left[S_{22}^a\right]\right]^{-1}\left[S_{12}^b\right] \tag{25}$$

$$\left[S_{21}^c\right] = \left[S_{21}^b\right]\left[[I] - \left[S_{22}^a\right]\left[S_{11}^b\right]\right]^{-1}\left[S_{21}^b\right] \tag{26}$$

$$\left[S_{22}^c\right] = \left[S_{21}^b\right]\left[[I] - \left[S_{22}^a\right]\left[S_{11}^b\right]\right]^{-1}\left[S_{22}^a\right]\left[S_{12}^a\right] + \left[S_{22}^a\right] . \tag{27}$$

Here, [*I*] is a unit matrix and [ ]$^{-1}$ represents the inverse matrix. The cascading process has the advantage that the exact number of junctions and sections does not have to be known at the start of the analysis as the process proceeds from the input to the output in a logical fashion [10].

The next stage is to determine the scattering matrices of the separate sections. These can be of two forms, either a uniform section or a junction between two uniform sections.

2.4 Number of Modes and Sections

Time taken to compute the input and output coefficients for the nonlinear taper depends on the number of modes and the number of sections. The time is proportional to the number of sections and approximately proportional to the square of the number of modes. The number of sections and the number of modes has to be judiciously taken so that the desired accuracy is achieved without unnecessary increasing computation time. The total number of modes depends on the relative diameters at each junction and must be chosen by repeat tests. The larger the change in diameter, the more modes will be excited locally and the higher the level of mode conversion.

2.5 Scattering Matrix Formulation for the Cylindrical Waveguide Nonlinear Taper

The total nonlinear circular waveguide taper length is divided into step discontinuity. Larger the number of sections taken, higher the accuracy in the analysis results obtained. The usual choice of the testing eigenmodes as being those of the smaller guide for enforcing continuity and as those of the smaller guide for enforcing the magnetic continuity at each junction is justified rigorously. The waveguide type can be arbitrary but the two waveguides must be identical to the cross section of the smaller waveguide. Also, all guides are required to posses the same axis. This simplifies the analysis since only modes with zero azimuthal variation ($TE_{0,n}$) need to be considered. As fields defined left of the junctions and right of the junctions as the sum of normal modes of respective waveguides. The set of equations (8)-(11) are used considering equal number of modes, *N* modes being present in both left and right sides of junction. Here, *N* is chosen large for convergence [11].

In equations (8)-(11) $e_{nL,R}$ and $h_{nL,R}$ are the normalized vector functions for the nth mode. In the circular waveguide, TE eigenmodes for the transverse electric fields can be calculated on either side of junction using the relation

$$e_{mn} = \frac{\sqrt{\varepsilon_m}}{\sqrt{\pi}\, J_m\left(\gamma'_{mn}\right)\left(\gamma'^{2}_{mn} - m^2\right)^{0.5}} \times \begin{bmatrix} \dfrac{m}{r} J_m\left(\dfrac{\gamma'_{mn} r}{a}\right)\sin(m\theta)(unit\, r) + \\ \left(\dfrac{\gamma'_{mn}}{a}\right) J'_m\left(\dfrac{\gamma'_{mn} r}{a}\right)\cos(m\theta)(unit\,\theta) \end{bmatrix}. \quad (28)$$

For our mode of interest $TE_{0,3}$, Substitute $m = 0$ and $n = 3$ for calculation of desired eigenmode. For m = 0, $\varepsilon_m = 1$ and $\gamma'_{mn} = 10.174$.

Also $$h_{mn} = z \times e_{mn} \quad . \quad (29)$$

Equation (29) can be rewritten as,

$$h_{03} = z \times e_{03} \quad . \quad (30)$$

For both side of junctions, the normalization of e_{Ln} and h_{Ln} using (17)

$$\int_{SL} \left(e_{Ln} \times h_{Ln}\right).ds = R_{nn}$$

and from the orthogonality of waveguide mode,

$$\int_{SL} \left(e_{Ln} \times h_{Ln}\right).ds = 0 \quad . \quad (31)$$

Matching the electric field and magnetic fields over the common apertures between the two regions:

$$E_L = E_R \text{ inside } S_L,\ E_L \times h_{Rn} = \mathbf{E}_R \times h_{Rn} \text{ inside } S_L \quad . \quad (32)$$

So, one can modify the equation (31) as,

$$\int_{S_L} \left(E_L \times h_{Rn}\right).ds = \int_{S_R} \left(E_R \times h_{Rn}\right).ds \quad . \quad (33)$$

Since, the electric field is null at the conductor surface, making up the surface S_R-S_L, the integral limit on the right hand side may be modified as

$$\int_{S_L} \left(E_L \times h_{Rn}\right).ds = \int_{S_R} \left(E_R \times h_{Rn}\right).ds \quad . \quad (34)$$

Using the properties,

$$\sum_{n=1}^{M} \left(A_n + B_n\right) P_{nn} = \left(C_n + D_n\right) Q_{nn} \quad (35)$$

where

$$P_{nn} = \int_{S_L} (e_{Ln} \times h_{Rn}).ds \text{ and } Q_{nn} = \int_{S_R} (e_{Rn} \times h_{Rn}).ds .$$

The other boundary condition required is, $H_L = H_R$ within S_L. Following

$$\int_{S_L} (e_{Ln} \times H_L).ds = \int_{S_R} (e_{Ln} \times H_R).ds .$$

With similar line of reasoning yields

$$R_{nn}(A_n - B_n) = \sum_{n=1}^{N} p_{nn}(C_n - D_n) \qquad (36)$$

$$R_{mm} = \int_{SL} (e_{Lm} \times h_{Lm}).ds .$$

Equations (35) and (36) may be rewritten into a matrix form as:

$$[P][A+B] = [Q][C+D] \quad , \qquad (37)$$

$$[R][A-B] = [P]^T [D-C] \quad . \qquad (38)$$

Above equation is converted into a scattering matrix format relating the normalized output vector B and D to the normalized input vector A and B. Matrices $[S_{11}]$, $[S_{12}]$, $[S_{21}]$ and $[S_{22}]$ are derived from $[P]$, $[P]^T$, $[R]$ and $[Q]$ matrices using matrix math and equations (37) and (38) as:

$$\left.\begin{aligned}
[S_{11}] &= \left[\sqrt{R}\right]\left([R]+[P]^T[P]\right)^{-1}\left([R]-[P]^T[P]\right)\left[\sqrt{R}\right]^{-1} \\
[S_{12}] &= 2\left[\sqrt{R}\right]\left([R]+[P]^T[P]\right)[P]^T[Q]^{-1} \\
[S_{21}] &= 2\left[\sqrt{Q}\right]\left([Q]+[P]^T[P]\right)[P]^T[R]^{-1} \\
[S_{22}] &= \left[\sqrt{Q}\right]\left([Q]+[P]^T[P]\right)^{-1}\left([Q]-[P]^T[P]\right)\left[\sqrt{Q}\right]^{-1}
\end{aligned}\right\} \qquad (39)$$

This is the calculation at single junction. So, calculation for transmission coefficient S_{21} is sufficient for our analysis. The cascading equation (26) is used only for S_{21} for two junctions *a* and *b*.

3. RESULTS AND DISCUSSION

A nonlinear tapered cylindrical waveguide often needed in the microwave/millimeter-wave system to connect the two waveguides of different radii. The problem in the design of these inhomogeneous waveguides is basically of specifying a distributed inhomogeneity for minimum mode conversion and/or minimum reflection over a specific frequency range. If the physical length of the device is at the same time kept at a minimum, the power loss will also be minimum. The presence of such a taper in a waveguide section inevitably introduces unwanted parasitic modes whereas one wishes that tapers should have a good match at the input port and prescribed spurious mode suppression at the output port keeping the taper length as short as possible. Various shape profile nonlinear tapers are used for the purpose. In order to analyse the performance of these nonlinear tapers various techniques are available.

In the present paper, we have adapted the mode matching field analytical technique which involves matching of the total modal field at each junction between uniform sections so that conservation of power is maintained. From this process, the amplitudes of the separate modes at the output of a junction have been deduced in terms of the amplitudes of the mode spectrum at the input to the junction. Total length of the taper is divided into a large number of section /junctions. Each junction along the length has its own scattering matrix. The matrices for all junctions have been be cascaded and an overall scattering matrix has been formed. The overall scattering matrix contains the input reflection coefficient and the output transmission coefficient for the inividual modes.

The developed modal matching technique has been applied for the 200kW 42GHz gyrotron oscillator under developement, with which our group is also involved. Here, a nonlinear tapered cylindrical waveguide section having a total length of 350.00 mm connects the cylndrical output section of the RF interaction cavity of radius 13.99 mm operating in the TE_{03} mode with the collector of radius 42.50 mm of the device.

Taking above taper dimension, nonlinear taper different shape profile, like, linear, parabolic, exponential and raised cosine have been analyzed for the transmission (S_{21}) and reflection coefficients (S_{11}) in the desired TE_{03} operating mode using mode matching technique.

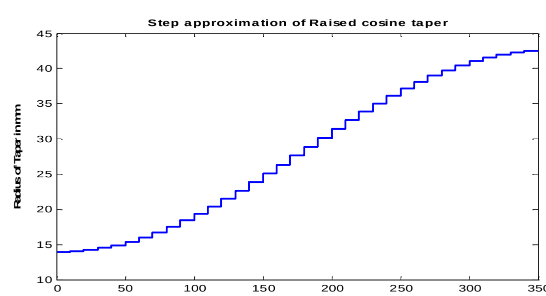

Figure 4: Step approximation of basic raised cosine function for analysis

Using the specific shape profile functions with the given input and output radii and length, taper counters are generated. Then, these contours which are smooth curve is approximated into the desired number of steps. Typically, figure 4 shows the step approximation of raised cosine function taper used for the analysis. The number of steps is increased in the analysis for converging results with desired level of accuracy. The number of sections taken is important issue. More number of steps taken, less the approximation, and better the transmission of power in every profile. Total power transmission in all modes is dependent on profile along with its dependency on the no. of sections. Profile of taper is responsible for mode conversion issue and also somewhat affected to the total power arrived at the output in all modes. After certain large enough sections, the mode conversion and transmission is saturated to certain value. Even increase in sections further remains those quantities unaffected. The transmission power is almost approaches to one while reflected power is very less as the waveguide is overmoded.

3.1. S-Parameter Analysis with Frequency Sweep

Analysis frequency range is taken from 40GHz to 45GHz. Transmission coefficient (S_{21}) is plotted with frequency range. Our concern is calculation and observation of S_{11} and S_{21} at 42GHz for each of basic design. Incident mode at input of taper is $TE_{0,3}$. S_{21} is total power transmitted at the output of taper retained in $TE_{0,3}$ mode. Transmission power calculation in desired mode is our objective**.**

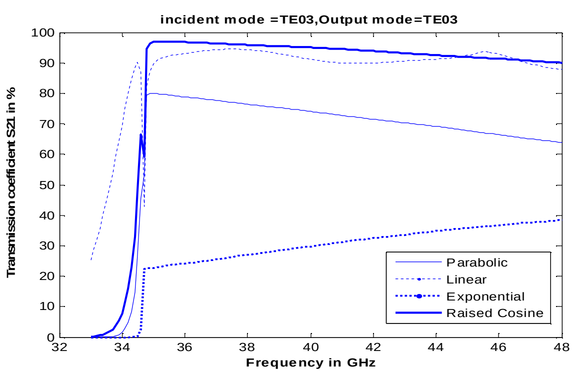

Figure 5: Transmission characteristics of basic profiles with frequency

From figure 5, one can observe the total transmission and reflection of the RF waves for the different shape profile taper designs for the desired operating $TE_{0,3}$ mode and gyrotron oscillation frequency of 42GHz. When the reflected waves are negligible in overmoded nonlinear waveguide then, total RF transmission in a desired mode can be taken corresponding to S_{21}. Table below showing the analytical transmitted power in desired $TE_{0,3}$ mode at output along with the transmission coefficient. The incident power in $TE_{0,3}$ mode is consider as unity.

Table 1: Basic Profile Transmission Coefficients and Transmitted power at 42 GHz.

Profile	S_{21}	Transmitted power at output in TE_{03} mode
Linear	89.9%	80.8%
Parabolic	71.5%	51.1%
Exponential	32.5%	10.6%
Raised Cosine	93.9%	88.1%

The analytical transmission coefficient for the desired operating mode among the four practical shape profile selected, the raised cosine nonlinear taper provides maximum transmission and minimum reflection. This result corroborates with the earlier findings reported in the literature [12].

4. CONCLUSION

A modal matching field analytical techniques have been developed for the analysis of arbitrrily shaped nonlinear waveguide tapers. The developed technique could easily be used to develop recursive computer codes for the desired accuracy. For a typical practical case, different type, like, linear, parabolic, expontial and raised cosine shaper profile have been taken and it have been found that the raised cosine shape profile nonlinear taper has been found most suitable since it has minimal shaper variation at both ends (input as well as output) thereby ensuring minimal reflections, with minimum transmission in the desired mode with appreciably low mode conversion.

It is hoped that the present analysis will be useful to the microwave practitioners in the analysis and design of optimum performance nonlinear tapers for its final contour selection and adaption in the high power microwave/millimeter wave devices and systems

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