ISSN 2319-2720

Volume 1, No.1, July – August 2012
International Journal of Computing, Communications and Networking
Available Online at http://warse.org/pdfs/ijccn05112012.pdf

Channel Capacity of MIMO System over NAKAGAMI- *m* Fading Channel

Suchismita Bhattacharjee Ph.D Scholar, NERIST, Arunachal Pradesh, India
getsuchi87@gmail.com
A. Dinamani Singh, Assistant Professor, NERIST, Arunachal Pradesh, India
ads@nerist.ac.in

ABSTRACT

Multiple Input and Multiple Output (MIMO) system have the potential to achieve very high capacities, depending on the propagation environment. A MIMO system provides multiple independent transmission channels, thus, leading (under certain conditions) to a channel capacity that increases linearly with the number of antenna elements. It is well known that the employment of multiple antennas at both the transmitter and receiver increases the overall system capacity. The objective of this paper is to simulate the Nakagami – *m* fading model using MATLAB and to analysis channel capacity. The program, when called in the MATLAB workspace, will generate random variables (RVs) that follow the Nakagami – *m* distribution. The generated random variable processes, includes the PDF and Nakagami PDF plots were drawn and which is suggested to apply this method to generate the Nakagami – *m* fading channel with MIMO Channel matrices by applying waterfilling algorithm using the same technique. For maximizing the capacity of MIMO systems, waterfilling algorithm is implemented by singular value decomposition (SVD) of received signal.

Keywords: MIMO, Gamma Random Variable, Waterfilling, SVD.

1. INTRODUCTION

Digital communication using multiple- input multiple output (MIMO) wireless links has recently emerged as one of the most significant technical breakthrough in modern communications. This paper presents an overview of some important theoretical concepts of MIMO systems. In the never-ending search for increased capacity in a wireless communication channel it has been shown that by using MIMO (Multiple Input Multiple Output) system architecture it is possible to increase that capacity substantially. Usually fading is considered as a problem in wireless communication but MIMO channels uses the fading to increase the capacity. In terms of queuing theory, it is known that gamma distribution arises naturally in processes for which waiting times between poison distributed events are relevant [1]. Multipath fading occurs in any environment where there is multipath propagation and there is some movement of elements, within the radio communications system. This may include the radio transmitter or receiver position, or in the elements that give rise to the reflections. The multipath fading can often be relatively deep, i.e. the signals fade completely away, whereas at other times the fading may not cause the signal to fall below a useable strength.[2].

2. MIMO CHANNEL MODEL

Multiple inputs multiple outputs (MIMO) systems in wireless communications refer to any wireless communication system where at both sides of the communication path more than one antenna is used. The idea behind MIMO is that, the signals on the transmit (TX) antennas at one end and the receive (RX) antennas at the other end are “combined” in such a way that the quality (bit-error rate or BER) or the data rate (bits/sec) of the communication for each MIMO user will be improved.[3].

A key feature of MIMO systems is the ability to turn multipath propagation, traditionally a pitfall of wireless transmission, into a benefit for the user. MIMO effectively takes advantage of random fading and when available, multipath delay spread for multiplying transfer rates. The prospect of many orders of magnitude improvement in wireless communication performance at no cost of extra spectrum (only hardware and complexity are added) is largely responsible for the success of MIMO as a topic for new research.

Principle: MIMO is based on main spatial multiplexing principle called Spatial Multiplexing. It is a transmission technique to transmit independent and separately encoded data signals, so-called streams, from each of the multiple transmit antennas. Therefore, the space dimension is reused, or multiplexed, more than one time [4].

In order to design efficient communication algorithms for MIMO systems and to understand the performance limits, it is important to understand the nature of the MIMO channel. For a system with M_T transmit antennas and M_R receive antennas, assuming frequency-flat fading over the bandwidth of interest, the MIMO channel at a given time Instant may be represented as an $M_R \times M_T$ matrix [6].

$$H = \begin{pmatrix} H_{1,1} & \cdots & H_{1,M_T} \\ \vdots & \ddots & \vdots \\ H_{M_R,1} & \cdots & H_{M_R,M_T} \end{pmatrix} \tag{1}$$

Where, M_r is the m^{th} term and M_t as n^{th}, so $H_{m,n}$ is the (single-input single-output) channel gain between the m^{th} receive and n^{th} transmit antenna pair. The nth column of **H** is often referred to as the spatial signature of the n^{th} transmit antenna across the receive antenna array. The relative geometry of the M_T spatial signatures determines the distinguish ability of the signals launched from the transmit antennas at a receiver.

3. CONCEPT OF NAKAGAMI FADING

With Nakagami-*m* distribution, sometimes denoted by *m*-distribution, a wide class of fading channel conditions can be modeled as explained in the introduction. This fading distribution has gained a lot of attention lately, since the Nakagami-*m* distribution often gives the best fit to land mobile and indoor mobile multipath propagation as well as scintillating ionospheres radio links.[11] More recent studies also showed that Nakagami-*m* gives the best fit for satellite-to-indoor and satellite-to-outdoor radio wave propagation [12].

Nakagami Fading occurs for multipath scattering with relatively larger time-delay spreads, with different clusters of reflected waves. Within any cluster, the phases of individual reflected waves are random, but the time delays are approximately equal for all the waves. As a result the envelope of each cluster signal is Rayleigh Distributed. The average time delay is assumed to differ between the clusters. If the delay times are significantly exceed the bit period of digital link, the different clusters produce serious intersymbol interference. Therefore the pdf (probability density function) is given as [2][6],

$$\rho_\gamma(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} \exp\left(\frac{-m\gamma}{\bar{\gamma}}\right), \gamma \geq 0 \qquad (2)$$

Where, $\bar{\gamma}$ is average fading power, $\gamma \geq 0$ as the channel amplitude, $\exp(\bullet)$ is the expectation operator, $\Gamma(\bullet)$ is the gamma distribution and m is the nakagami fading parameter.

4. CONCEPT OF WATERFILLING THEOREM

Waterfilling is a metaphor for the solution of several optimization problems related to channel capacity. The simplest physical example is perhaps the case of spectral allocation for maximal total capacity under a total power constraint[10].

Many engineering problems that can be formulated as constrained optimization problems result in solutions given by waterfilling structure the classical example of which is the capacity achieving solutions for the MIMO channel. The problem of jointly designing the transmitter and the receiver for communication through MIMO channel also results in a waterfilling solution. The well – known classical waterfilling solution solves the problem of maximizing the mutual information between the input and the output of a channel composed of several sub channels (such as a frequency – selective sub channels arising from the use of multiple antennas at both sides of the link)[3][10] with a global power constraint at the transmitter.

This capacity – achieving solution has the visual interpretation of pouring water over a surface given by the inverse of the sub channel gains hence the name waterfilling or waterpouring.

a. Waterfilling capacity of MIMO channel

When the channel knowledge is absent at the transmitter, the individual sub channels are not accessible. So the equal power allocation in all the sub channels is logical under this scenario. When the transmitter has perfect knowledge of the channel, the waterfilling method theorem so the division of total power in such a way that a greater portion goes to the sub channels with higher gain and less or ever none to the channels with small gains.

The sub channels with lower gain i.e those with higher noise for which no power is allocated at all refer to those sub channels which are not used for transmitting any signal during the transmission. One objective of this algorithm is to allocate power across the channel so as to mazimize the total capacity.

This power allocation is subject to the constraint that the sum of the power poured into all sub channels is equal to P_T, the total power available to the transmitter.

The relative channel strengths and the amount of power to allocate to each channel is determined by knowledge of the channel matrix, H.

We use the eigen decomposition of H to obtain as,

$$H(r-by-t) = UDV^+; \qquad (3)$$

Where, $UU^+ = I_r = VV^+ = I_t$ and $D = diagonal \lambda_1, \lambda_2, ...\lambda_n$ with λ_i as the positive square root of i^{th} eigen value and i= 1 to n non zero λ values and $n = \min\{r,t\}$. [15] $H = UDV^+$

The first step is to determine the parameter μ. The parameter μ, is a mathematical parameter, used to determine the power assigned to each of the sub channels of the composite MIMO channel. After determining the μ, the square of the inverse of eigen values are compared with μ.

If the square of the inverse of i^{th} eigen value is greater than μ, i.e if $1/\lambda^{-2} \geq \mu$, then that i^{th} eigen channel is too weak to be used for the communication process. The last two sub channels in the above illustrated example of a (7 – by – 7) MIMO channel are such eigen channels which are not used for transmitting any signal at that point of time.

Such channels are said to be switched off and they are put away from the communication process which means that those particular sub channels are not allocated with any transmitting power.

Once the total available power, P_T and the gains of the parallel sub channels are known, the optimum power allocated to the i^{th} sub channel is,

$$p_i = \left(\mu - \frac{1}{\lambda_i^2}\right)+^{\bullet} \qquad (4)$$

If this quantity $p_i = \left(\mu - \frac{1}{\lambda_i^2}\right)$ is positive then the power is allocated to the i^{th} sub channel otherwise, the sub channel is left unused. The waterfilling parameter 'μ' is determined iteratively by the total power p_t , such that μ satisfies the following equation,

$$p_T = \sum_{i=1}^{m}\left[\mu - \frac{1}{\lambda^2}\right] \qquad (5)$$

I= 1,2,…..m; where m is the number of sub channels that have survived after checking the above conditions and are to be used for transmission of the signal.

Now the capacity of MIMO channel with waterfilling can be expressed as,[3]

$$C = \sum_{1}^{m}\log_2\left[1+\left(\frac{\rho_i}{\sigma^2}\right)*\rho_i^2\right] \text{ bps / Hz} \qquad (6)$$

Above equation enables the visualization of the MIMO channel as a number of parallel SISO pipes with gain equal. Therefore to the respective eigen values and its enables as to understand that the waterfilling capacity for MIMO channels is the sum of the capacities of the SISO equivalent parallel sub channels, obtained from performing SVD on MIMO channel matrix. If the channel is known at the transmitter, the capacity can be enhanced by using the good channels i.e. those with the highest gain by applying an unequal power distribution.

b. Step to power allocation with waterfilling alogorithm

Summary of steps involved in the waterfilling power allocation to the MIMO subchannels [7]:

1. The first step is to determine the waterfilling parameter or threshold, μ which is also shown as water level. The μ is just a mathematical parameter used to determine the power allocated to each of the eigen channels.
2. After determining μ, the inverse of eigen values of the matrix H is compared with the threshold.
3. Now if $1/\lambda^2 \geq \mu$ then, the gain of the i^{th} eigen channel is too small and this eigen channel will not be considered for communication, the last two eigen channel.
4. Assuming the case of a square dimension of MIMO channel, i.e. r=t and also $\lambda_1 \geq \lambda_2 > \lambda_3 \ldots\ldots \geq \lambda_n$. And also consider that m eigen values have survived after the above described procedure.
5. Once the total available power, P_T and the gains of the parallel sub channels are known, the optimum power allocated to the i^{th} sub channel is

$$\mu_i = \left[\mu - \frac{1}{\lambda_i^2}\right] \qquad (7)$$

And the power allocated to each of these eigen channels, P_i is determined by the waterfilling rule such that the above equations are satisfied. When it is positive then the i^{th} sub channel otherwise, the sub channel is left unused. The waterfilling parameter 'μ' is determined next part. Under ideal conditions, the information theoretic capacities of a MIMO system grows linearly with the minimum of transmit and receive antennas. However, various measurements show that realistic MIMO channels show a significantly lower capacity. This reduction of capacity is due to the spatial correlation of the MIMO channel coefficients. But here with waterfilling algorithm the capacity increases with correlation as proved before [8] as MIMO capacity analysis with respect to linear increment in number of transmitting and receiving antennas for fixed values of SNR with water filling power allocation algorithm assumed.

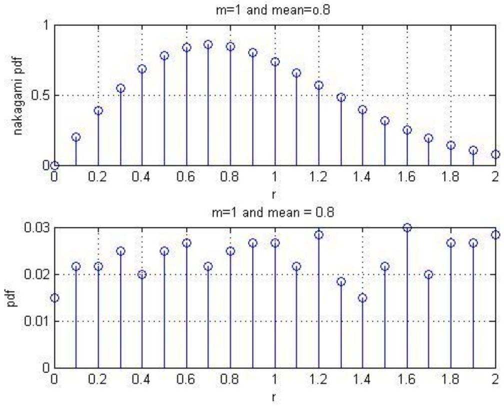
5. RESULT AND DISCUSION

MATLAB Simulation

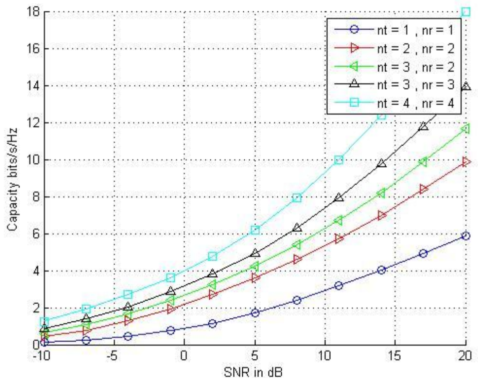
The capacity of the MIMO channel has been simulated for various number of transmitter and receiver antennas using the water- filling algorithm for allocation of the optimum power to the parallel sub channels, represented by the diagonal elements of the diagonal matrix which was obtained by performing the singular value decomposition on the MIMO channel matrix,

Channel capacity for Nakagami faded MIMO channel is considered with implementation of waterfilling. The graph of capacity Vs SNR shows that the capacity of the MIMO channel increases as the number of antennas used at both the transmitter and receiver increases.

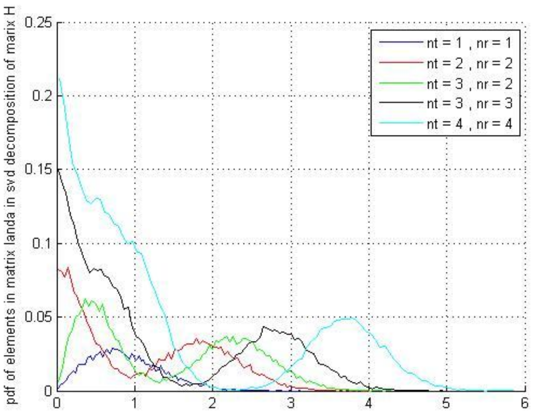
The result obtained shows that there is an improvement in capacity of MIMO channel when the waterfilling solution is implemented to achieve capacity maximization is used to allocate different powers to the sub channels. And for correlated Nakagami fading on MIMO channel with water-filling algorithm by decomposition theorem for capacity performance analysis, the MIMO capacity with respect in increase in number of transmitting and receiving antenna for fixed values of SNR with waterfilling power allocation algorithm assumed. The figure clearly indicates the incremental enhancement with different fixed values of SNRs. As comparison of average power allocation and waterfilling allocation of power in sub channels for MIMO capacity analysis with different subsets of transmit and receive antennas. Here, the power allocation with individual matrix element “landas” for different antenna selections is being plotted. As with rician fading it was confirmed that the use of MIMO system with correlation increases the capacity, here it is being proved that with nakagami also the use of MIMO system gives greatly increase in the achievable rate (capacity) with certain degree of correlation.



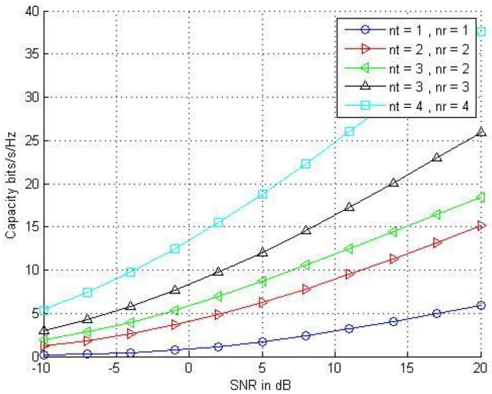
Random variable for Nakagami distribution: for normal pdf and nakagami pdf.



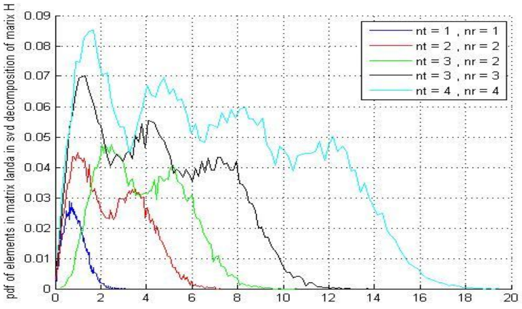
Channel Capacity for uncorrelated MIMO Nakagami – *m* fading with Waterfilling Algorithm



PDF of elements in matrix landa in SVD decomposition of matrix H



Channel Capacity for correlated MIMO Nakagami – m fading with Waterfilling Algorithm



PDF of elements in matrix landa in SVD decomposition of matrix H

6. CONCLUSION AND FUTURE SCOPES

This paper mainly gives investigations on capacity of MIMO channel. The waterfilling algorithm was implemented in this work to carry out simulations on the MIMO channel capacity and the analysis of the Nakagami – *m* signal fading model in wireless communication, through multipath propagation channels. By generating random variable for Nakagami – m distribution or Nakagami random variable is plotted and implemented on MIMO system with Waterfilling Algorithm for SVD method and decomposition method for plotting channel capacity and pdf of element of antenna in eigen value matrix for matrix H for capacity analysis for uncorrelated and correlated forms where in both cases the capacity increases with increase in number of transmitter and receiver, which is being observed and justified.

In future scope, as the idea of single user power optimization in a single user MIMO system can be extended to a multi-user MIMO system.

REFERENCES

1. Nelson Coster and Simon Haykin. **MIMO (Multiple Input Multiple Output) - Theory and Practice**, Wiley Publications, Chapter 1,2,3.

2.Marvin K. Simon, Mohamed-Slim Alouini. **Digital Communication over Fading Channels: A Unified Approach to Performance Analysis**, Copyright 2000 John Wiley & Sons, Inc. Print ISBN 0-471-31779-9 Electronic ISBN 0-471-20069-7, Chapter 4 and Chapter 9.

3. M. Kusuma, M. Amgothu and B. Amgothu. **Capacity Analysis of MIMO systems – using waterfilling algorithm**, International Journal of Advances in Computer Networks and its Security.

4. Sergey Loyka. **MIMO channel capacity: Electromagnetic wave perceptive**, School of Information Technology and Engineering (SITE), University of Ottawa.

5. Zhang Q.T. **Correlated Nakagami Fading Channels with different Fading Parameters: A Generic Characterization with Application**, Communications 2002. Icc 2002. IEEE international conference on volume 2, pp.: 704- 708, Vol.2, 28 April- 2 May 2002.

6. Yong Soo Cho, Jaekwon Kim, Won Young Yang and Chung Gu Kang. **MIMO-OFDM Wireless Communications with MATLAB**, IEEE PRESS, John Wiley & Sons (Asia) Pte Ltd., Chapter 1 and Chapter 3.

7. Iskander,C.D. and Takis Mathiopoulas P. **Analytical Level crossing rates and average Fade durations for Diversity Techniques in Nakagami Fading Channels**, Communications, IEEE transactions on Volume 50, Issue 8, pp: 1301 – 1309, Aug. 2002.

8. Shreedhar A. and Jhoshi el. Al. **Modeling and capacity analysis of correlated MIMO channels**, International Journal of Engineering Science and Technology, Vol.2(10), pp-5419 – 5423, 2010.

9. Hilde Skjevling, **Simulation and design of MIMO algorithm for Correlated Wireless Channels**", University of Oslo, Dept. of Informatics, 25th July 2003.

10. Kuldeep Kumar and Manwinder Singh. **Capacity Enhancement of MIMO system using Water filling Model**, (IJAEST) International Journal of Advanced Engineering Sciences and Technologies, Vol No. 7, Issue No. 1, pp. 92 – 97.

11. Elina Pajala, Tero Isotalo, Abdelmonaem Lakhzouri and Elena Simona Lohan. **An improved simulation model for Nakagami-*m* fading channels for satellite positioning applications**, Proceedings of the 3rd Workshop on Positioning, Navigation and Communication (WPNC'06).

12.Cuifen Cheng. **A Nakagami – *m* fading channel simulator,** November 2000.

13. Venugopal V. and Veeravali. **Correlated MIMO wireless channels: capacity, optimum signaling, and asymptotics,** IEEE transactions on Information Theory, Vol. 51, No. 6, June 2005.

14. Claude Oestges and Bruno Clerckx. **MIMO: Wireless Communications - from real – world propagation to space – time code design**, Introduction to Multi- Antenna Communication.