

## Recursive Least Square Technique For Channel Estimation For Transmit Diversity Case In MIMO-OFDM

Dhariti Sharma

Department Of Electronics And Communication Engineering, Bahra University,

Shimla-Hills, H.P, India

Dharitis@Gmail.Com

### ABSTRACT

*The traffic in wireless networks has been showing an exponential growth over the last decade. In order to meet the demand, and support a continuation of this growth, the scarce radio resources need to be efficiently used. The use of MIMO combined with OFDM systems has significantly improved the reliable system performance. Combination of multiple-input multiple-output (MIMO) system with orthogonal frequency division multiplexing (OFDM) technique can be exploited to attain high data rate and better spectral efficiency. Channel estimation plays a major role in communication system. In mobile communication systems bits of information is transmitted by making changes in amplitude or phase of radio waves. Channel estimation is the estimation of transmitted signal bits. The paper proposes an implementation of Recursive Least Square (RLS) technique for the Channel estimation in Transmit Diversity.*

### Keywords

*Channel estimation, MIMO, OFDM, LS, RLS*

### 1. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a popular modulation scheme that is used in wireless LAN standards like 802.11a, g, HIPERLAN/2 and in the Digital Video Broadcasting standard (DVBT) [1]. OFDM systems can provide a composite high-data-rate with a long symbol duration. That helps to eliminate the ISI (intersymbol interference), which often occurs along with signals of a short symbol duration in a multipath channel [1]. Channel estimation plays an important part in an OFDM system. The bits of information are transmitted by creating changes in amplitude or phase of radio waves in mobile communication system. Amplitude or phase varies considerably on the receiving side. The presentation of receiver is extremely dependent on the correctness of the predictable immediate channel so as a

consequence system excellence is tainted [2]. Appropriate to this motivation channel estimation technique is introduced so that the correctness of the received signal is enhanced. The radio channels are typically multipath fading channels which causes Inter Symbol Interference (ISI) in the received signal. A divide channel estimator provides information on channel impulse response [2]. The channel estimator is based on the known succession of bits which are single for certain transmitter. Thus channel estimator estimates CIR separately for each burst from the known transmitted bits.

OFDM is a multicarrier intonation method used to transmit high rate data stream all the way through wireless medium. A high rated data stream is divided into parallel lower rate data streams which are transmitted at the same time over a number of separate subcarriers. This technique also eliminates Inter Symbol Interference [3]. In OFDM, implementation is performed by making use of Fast Fourier Transform (FFT)/Inverse Fast Fourier Transform (IFFT) algorithms. It improves the frequency spectral efficiency and minimizes the complexity of the receiver by converting the frequency selective channel into collection of parallel frequency flat sub channels [3].

MIMO (Multiple Input Multiple Output) makes use of multiple antennas at the transmitter and the receiver. In modern usage, "MIMO" specifically refers to a practical method for sending and receiving more than one data signal on the same radio channel at the similar time via multipath circulation. The use of multiple antennas at both sides of the wireless link provide a most promising solution to enhance the bandwidth efficiency and reliability of system without any need of extra bandwidth or transmitting more power to the channel. [4]

The objective of this paper is to present an efficient MIMO channel estimation technique with less Bit error rate and Mean square error with less complexity. To reduce the complexity of the estimation, the MIMO channel is considered as a superposition of several multiple-input single-output (MISO) channels [4]. Also a multicarrier technique OFDM is used which increases the data rate. The paper is organized as follows. The fundamentals of MIMO-OFDM systems are described in section II. In Section III the Least Square channel estimation and Adaptively Regularized Least Square estimation is shown. Recursive Least Square estimator is described in section IV. The performance of the MIMO-OFDM system is optimized with minimum bit error rate and mean square error. Section V presents the simulations and results. Conclusion marks can be found in Section VI.

## 2. MIMO-OFDM SYSTEMS

Conventional wireless communication systems provided constant bandwidth, no option of growing the sending rate of information. Bandwidth, information sending rate and software-hardware complexities are the important parameters to design a communication system [4]. Methods such as MIMO, OFDM and integrating them as MIMO-OFDM are optional to make bigger the new generation of communication system.

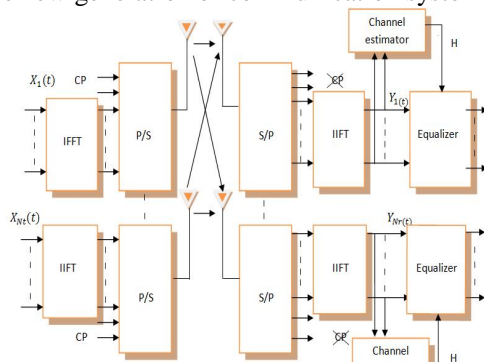


Figure 1 MIMO communication system with  $N_T$  transmitting antennas and  $N_R$  receiving antennas

The MIMO-OFDM system as shown in Figure 1 has  $X_1$  transmitting antenna,  $X_{Nt}$  receiving antennas and  $K$  subcarriers. We suppose the time-varying wireless is quasi-static with maximum multipath delay length of  $L$  [4]. Suppose cyclic prefix of OFDM symbol is chosen to be longer than  $L$ .

Channels between all transmit and receive antenna pair are independent. The MIMO-OFDM transmitter has  $N_t$  parallel transmission paths and each path perform serial-to-parallel conversion, pilot insertion, IFFT and cyclic extension before the final TX signals are up converted to RF signal and transmitted. The channel encoding and digital modulation can also be performed per branch, where the modulated signals are space-time coded using the Alamouti algorithm before transmitting from multiple antennas. At the receiver side the CP is removed and  $N$ -point FFT is performed per receiver branch and then the operations like digital demodulation and decoding are performed. Finally all the input binary data are recovered with some BER.

## 3. CHANNEL ESTIMATION ALGORITHMS

The key and important task is channel estimation in coherent communication systems because it is a main issue for coherent OFDM systems [5]. Channel estimation is more difficult because of greater number of channels to be predicted as compared to SISO systems. Channel estimation method is based on the signals association and MAI repression. The excellence of the channel inference method has a strong impact on the overall bit error rate (BER) presentation of the receiver [6].

The two types of channel estimation schemes are, (1) Pilot assisted schemes, in which a portion of the bandwidth is assigned to training symbols and, (2) Blind approach, which can be implemented by using statistical properties. At receiver side, channel estimation can be done by adding pilot signals into the transmitted signals. Pilot tones, along with OFDM symbol are used for estimating the channel [7]. Block type and comb type pilot arrangements are the two types of pilot channel insertions.

### 3.1 Least square Estimation

LS estimator minimizes the following cost function [8]

$$\min_{\hat{h}} (Y - X\hat{h})^H (Y - X\hat{h}) \quad (1)$$

where  $[.]^H$  is the Hermitian (conjugate) transpose operator. Then, the LS estimation of  $h$  is given by

$$\hat{h}_{LS} = \frac{Y}{X} = \begin{bmatrix} Y_k \\ X_k \end{bmatrix}^T \quad (2)$$

This LS estimator is equivalent to what is also referred to as the zero-forcing estimator since it can also be obtained from the time domain LS estimator with no assumption on the number of CIR taps or length. That is

$$\hat{h}_{LS} = FQ_{LS} = F^H X^H Y \quad (3)$$

$$Q_{LS} = (F^H X^H X F)^{-1} \quad (4)$$

Note that this easy LS estimator does not exploit the correlation of channel across subcarriers in frequency and across the OFDM symbols in time. Without using any information of the statistics of the channel, the LS estimator can be calculated with very low complication, but it has a high mean-square error since it does not take into account the effect of noise on the signal [8].

### 3.2 Adaptively Regularized Least Square Estimator

In transmit-diversity OFDM systems, DDCE demand high computational complexity due to inter-antenna interference (IAI) and the need for simultaneous estimation of all channel responses. In transmit diversity OFDM systems the performance of DDCE using LS estimation is degraded by the data of incorrect decisions [9].

An adaptively regularized LS estimator was proposed in [9] to improve the performance of DDCE in transmit-diversity OFDM systems. The regularized LS estimator includes a priori channel information in calculating the new channel estimate. The latest DDCE estimate is adopted as a priori channel information by considering temporal channel correlation. Furthermore, the regularization parameter was designed to consider the modulus of IAI-cancelled data which are not equal and adaptively determined according to the mean squared error (MSE) of the latest DDCE estimate and that of the current standard LS estimate [9].

Defined next are

$$\sigma_{LS}^2(n) = E \|h_1[n] - \hat{h}_1^{LS}[n]\|^2 \quad (5)$$

And

$$\sigma_h^2(n) = E \|h_1[n] - \hat{h}_1[n-1]\|^2 \quad (6)$$

The variable  $\alpha_n$  is determined by solving for the minimum of the MSE in according to the following equation [9]:

$$\alpha(n) = \beta \cdot \sigma_{LS}^2(n) / \sigma_h^2(n) \quad (7)$$

Thus, the following equation for approximating the regularized least-squares estimate was formulated:

$$\hat{h}_1[n] = \frac{\sigma_h^2}{\sigma_h^2 + \alpha(n)} \hat{h}_1^{LS}[n] + \frac{\alpha(n)}{\sigma_h^2 + \alpha(n)} \hat{h}_1[n-1] \quad (8)$$

$\alpha(n)$  can be adaptively determined using

$$\alpha(n) = \sigma_v^2 \sigma_{LS}^2(n) / \sigma_h^2(n) \quad (9)$$

## 4. RECURSIVE LEAST SQUARES ESTIMATION

We achieve measurements sequentially and want to update our estimate with every new measurement. In this case, the matrix  $H$  requires to be augmented. In this, we have to compute an estimate  $\tilde{x}$  according to every newest measurement. Apart from that, the update may become much expensive. All the computation ought to be prohibitive as number of the measurements becomes large [11].

It displays how to recursively compute weighted least squares estimate. Moreover, assume an estimate  $\tilde{x}_{k-1}$  after the  $k-1$  measurement and attain a new measurement of  $y_k$ . Generally, every measurement is an  $m$ -vector with the values yielded by various measurement instruments. A linear recursive estimator can be written in the following form:

$$y_k = H_k x + v_k \quad (10)$$

$$\tilde{x}_k = \tilde{x}_{k-1} + K_k (y_k - H_k \tilde{x}_{k-1}) \quad (11)$$

Here  $H_k$  is the  $m \times n$  matrix,  $K_k$  is  $n \times m$  can be referred to the *estimator gain matrix*. We refer to  $(y_k - H_k \tilde{x}_{k-1})$  as the *correction term*. In this, new estimate  $\tilde{x}_k$  can be modified with the previous estimate  $\tilde{x}_{k-1}$  with a correction making use of a gain vector. In this, measurement noise has zero mean, i.e.,  $E(v_k) = 0$ .

The current estimation error is

$$\epsilon_k = x - \tilde{x}_k \quad (12)$$

$$\begin{aligned}
&= x - \tilde{x}_{k-1}K_k y_k - (H_k \tilde{x}_{k-1}) \\
&= \epsilon_{k-1} - K_k(H_k x + v_k - H_k \tilde{x}_{k-1}) \\
&= \epsilon_{k-1} - K_k H_k(x - \tilde{x}_{k-1}) - K_k v_k \\
&= (I - K_k H_k)\epsilon_{k-1} - K_k v_k
\end{aligned}$$

where I is the  $n \times n$  identity matrix. The

mean of this error is then

$$E(\epsilon_k) = (I - K_k H_k)E(\epsilon_{k-1}) - K_k E(v_k) \quad (13)$$

If  $E(v_k) = 0$  and  $E(\epsilon_{k-1}) = 0$ , then  $E(\epsilon_k) =$

0. So if the measurement noise  $v_k$  has zero mean for all  $k$ , and the initial estimate of  $x$  is put equal to its expected value, then  $\tilde{x}_k = x_k$  for all  $k$ . With this property, the estimator is called *unbiased*. [11]

#### 4.1 The Estimation Algorithm

In the following the algorithm used for recursive least squares estimation has been summarized [11]:

1. Initialize the estimator:

$$x_0 = E(x) \quad (14)$$

$$P_0 = E((x - \tilde{x}_0)(x - \tilde{x}_0)^T) \quad (15)$$

In this case no prior knowledge is required about  $x$ , just let  $P_0 = \infty I$ . In the case of perfect prior information, let  $P_0 = 0$ .

2. Iterate in the following two steps.

(a) Get the newest measurement  $y_k$ , assuming it is given by equation

$$y_k = H_k x + v_k \quad (16)$$

In this, noise  $v_k$  has zero mean or covariance  $R_k$ . Measurement noise at every time step, where  $k$  is independent. So,

$$E(vv^T) = \begin{cases} 0, & \text{if } i \neq j, \\ R_j, & \text{if } i = j \end{cases} \quad (17)$$

Eventually, we assume the white measurement noise.

(b) Update estimate  $\tilde{x}$  and the covariance of estimation error sequentially, that is re-listed below:

$$K_k = P_{k-1} H_k^T (H_k P_{k-1} H_k^T + R_k)^{-1} \quad (18)$$

$$\tilde{x}_k = \tilde{x}_{k-1} + K_k (y_k - H_k \tilde{x}_{k-1}) \quad (19)$$

$$P_k = (I - K_k H_k) P_{k-1} \quad (20)$$

## 5. SIMULATION AND RESULT

The performance of the MIMO-OFDM system is optimized with minimum bit error rate and mean square error. These algorithms can be applied to the AWGN, Rayleigh and Rician channel models. Then comparing the BER and MSE value for different low and high SNR value are implemented by using LS, Adaptively Regularized Least square estimator and RLS estimator.

For the simulation of basic MIMO-OFDM system, the following parameters are used as shown in Table 1.

Table 1. Simulation parameters for MSE and BER plots for AWGN, Rayleigh and Rician channel

Parameters	Specifications
Number of transmitters	2
Number of receivers	1
FFT size	64
Number of subcarriers	64
Signal constellation	QAM
Channel model	AWGN, Rayleigh, Rician
Cyclic prefix length	16

### 5.1 Simulation for AWGN channel

### 5.2 Simulation for Rician Channel

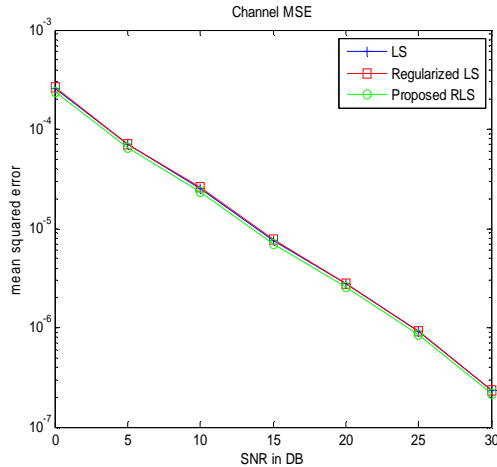


Figure 2 MSE graph for LS, Adaptively Regularized LS, Proposed method in AWGN channel

Figure 2 represents the SNR vs MSE plot for AWGN channel using LS, Adaptively Regularized LS, proposed method. The performance comparison shows that, the proposed method outperforms the previous estimation techniques in MSE performance.

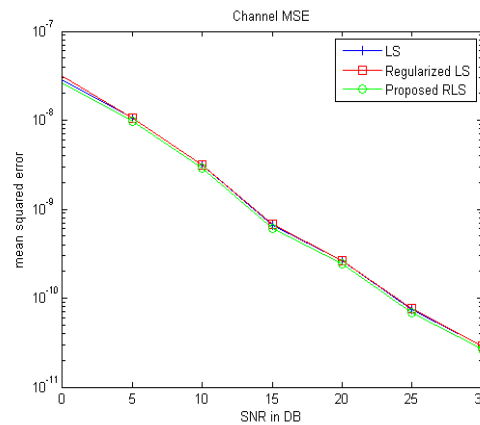


Figure 4 MSE curve for LS, Regularized LS and proposed RLS in Rician channel

Figure 4 represents the SNR vs MSE plot for Rician channel using LS, Adaptively Regularized LS, proposed method. The simulation results show that, the proposed method outperforms the previous estimation techniques in MSE performance.

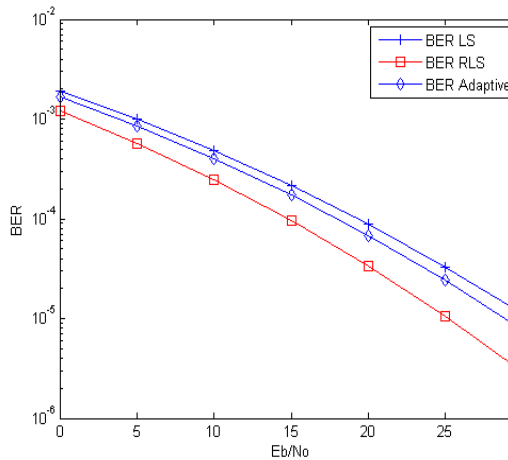


Figure 3 BER graph for LS, Adaptively Regularized LS, Proposed method in AWGN channel

Figure 3 represents the SNR vs BER plot for AWGN channel using LS, Adaptively Regularized LS, proposed method. The simulation results show that, the proposed method outperforms the previous estimation techniques in BER performance.

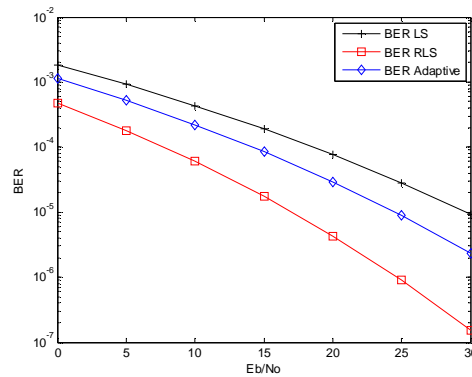


Figure 5 BER graph for LS, Regularized estimator and proposed method in Rician channel

Figure 5 represents the SNR vs BER plot for Rician channel using LS, Adaptively Regularized LS, proposed method. The simulation results show that, the proposed method outperforms the previous estimation techniques in BER performance.

### 5.3 Simulation for Rayleigh Flat fading

#### Channel

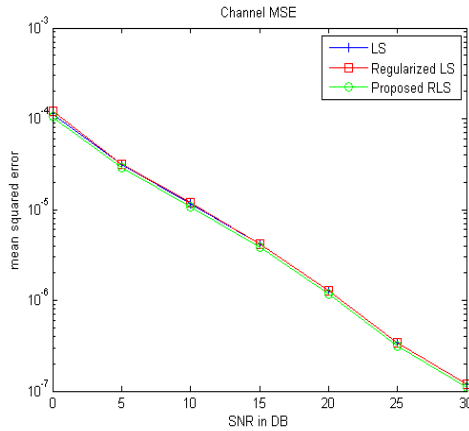


Figure 6 MSE curve for LS, Regularized LS, Proposed RLS in Rayleigh Flat fading channel

Figure 6 represents the SNR vs MSE plot for Rayleigh Flat fading channel using LS, Adaptively Regularized LS, proposed method. The simulation results show that, the proposed method outperforms the previous estimation techniques in MSE performance.

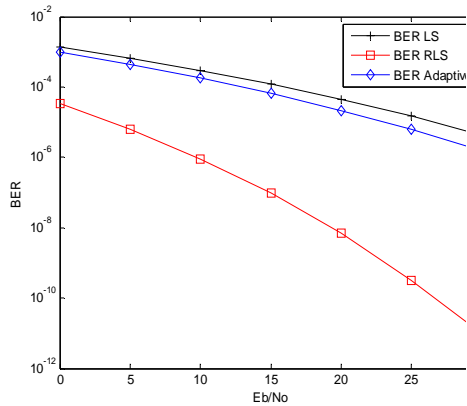


Figure 7 BER graph for LS, Regularized estimator and proposed method in Rayleigh Flat fading channel

Figure 7 represents the SNR vs BER plot for Rayleigh Flat fading channel using LS, Adaptively Regularized LS, proposed method. The simulation results show that, the proposed method outperforms the previous estimation techniques in BER performance.

### 6. CONCLUSION

In this paper, the various channel estimation techniques for MIMO-OFDM are studied. Channel estimation algorithms have been compared and results show that least square algorithm is the simplest amongst all but has low performance. The adaptively regularized method improves upon the existing LS technique and implements a slightly complex method. The proposed system in turns has similar complexity as that of the adaptively regularized method but has performed better than both the methods. Hence the proposed method can be considered for practical implementation.

The future work may involve the use of such methods which may further reduce the complexity of the algorithm. As can be observed from the results, the BER for both base and the proposed technique is almost zero, we can now focus on the reduction of complexity. Such a system may be lesser expensive in terms of hardware implementation. Hence, a system can be developed to further improve over the proposed method.

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#### Authors Biography

**Dhariti Sharma** received the B.Tech. degree from the Department of Electronics and Telecommunications, Eternal University Sirmaur, Himachal Pradesh in 2013 and is pursuing M.Tech degree in Electronics and Telecommunications, Bahra University, Solan. Her research interests include the areas of communications and signal processing, estimation and detection theory.

