A Novel Framework for Simulation of Quantum Information System

Binh A. Nguyen[1], Viet Q. Tran[1], Khoa D. Ta[1], Manh Hoang[1],Thien V. Truong[1], Nhan D. Nguyen[2], Duc M. Nguyen[3]

[1]ICT Department, FPT University, Hanoi, Vietnam, binhnase04865@fpt.edu.vn, viettqse06178@fpt.edu.vn, khoatdhe130813@fpt.edu.vn, manhhhe130294@fpt.edu.vn, thientvse04522@fpt.edu.vn
[2]Dept. of Biomedical Engineering, Sungkyunkwan University, Suwon, South Korea, nhannd@skku.edu
[3]School of Electrical Engineering, University of Ulsan, Ulsan, Korea, nguyenmanhduc18@gmail.com

ABSTRACT

Nowadays, quantum computers have been proven to solve the complex problem more efficient and process data more security than classical computer. However, the effect of noises and unwanted environments make the needs of quantum error correction code to build the practical quantum information system. Since the quantum computation algorithms need to be verified in classical computer before implement in quantum computers, the novel framework for verification in classical computer has been become essentially and importantly. In this discussion, a novel framework for quantum information system simulation is proposed which is based on **MATLAB** environment. First, we analysis the basic notation and elementary elements of a quantum system via mathematical model. Then, we introduce the basic elements of Pauli channel model, and two basic quantum error correction codes to stabilize the quantum channel. Hence, the proposed system to simulate the quantum three-repetition codes and quantum Shor codes are discussed.

Key words: Quantum computation, Quantum circuit model, Quantum error correction codes, Shor codes.

1. INTRODUCTION

An effort to make a generalization of classical computer proposes a new type of computer named quantum computer which are based on quantum mechanism. The computation algorithms based on quantum computer have proved the efficient on processing data more security and solving complex problem more efficient time [1]. For example, one of the first quantum algorithm to factor an integer into its primers is invented by Shor[2], which runs on polynomial time. Moreover, Grover [3] proposed a searching algorithm named Grover search, which is applied on many reality researches on large database system. Since then, quantum computers are attracted by many researchers all over the world [4-6, 16-20].

However, as an efficient computation system, quantum computers have faced the imperfectly applied of quantum channel noise, decoherence which affect the practical design of quantum computers. To overcome such those, an emerging technique from classical error correction codes is used to protect quantum computers, which is called quantum error correction codes (QECC). Since the first discussion of quantum code is invented by Shor [7], the theory of QECC is generalization to be expressed as the quantum stabilizer code [8]. Then, there are many proposed quantum codes for various purposes of practical design [9-11].

Before implementing the error correcting codes on quantum computer, the necessary step is to simulate them on the classical computer. There are many possible quantum computation models such as quantum circuits model, quantum adiabatic computation, Zidan's model [12-15], they are proven to have those effective on simulation of quantum algorithm, quantum protocol, quantum communication. To verify quantum error correction code, quantum circuit model is a suitable choice since it also based on mathematic model of Hilbert space like QECC. Hence, in this work we propose the framework based on **MATLAB** environment to simulate and analysis two basic QECC codes.

The paper is organized as follows. In Section 2, we review about the qubits, elementary operations of qubits, and quantum operations. The quantum error correction and the solution to simulate in **MATLAB** are discussed in Section 3. Finally, the conclusion is listed in Section 4.

2. ELEMENTARY OPERATIONS OF QUANTUM INFORMATION SYSTEM

Quantum theory uses qubit to represent information, the quantum systems with two levels such as: two polarization states of photons, two energy levels of atoms, etc. A qubit $|\psi\rangle = a|0\rangle + b|1\rangle$ is considered to be found in the both basis states $|0\rangle$ and $|1\rangle$ where the probability value we saw that qubit at state $|0\rangle$ is $|a|^2$ and at state $|1\rangle$ is $|b|^2$. It is superposition concept of a qubit, which is one of a main property of quantum information since the amount of information which presented in qubits are no limitation. A qubit can be displayed in matrix form as,

$$|\Psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix} = a|0\rangle + b|1\rangle. \qquad (1)$$

According to norm condition for a qubit on the Bloch sphere space, the complex numbers a and b satisfy the equation $|a|^2 + |b|^2 = 1$. A n qubits system is constructed by multiple tensor products of some other qubits, it is given as follows,

$$|\phi\rangle = \sum_{i=0}^{2^n-1} a_i |i\rangle = \sum_{i_k=\{0,1\}} a_{i_1 i_2 \ldots i_n} |i_1\rangle |i_2\rangle \ldots |i_n\rangle. \qquad (2)$$
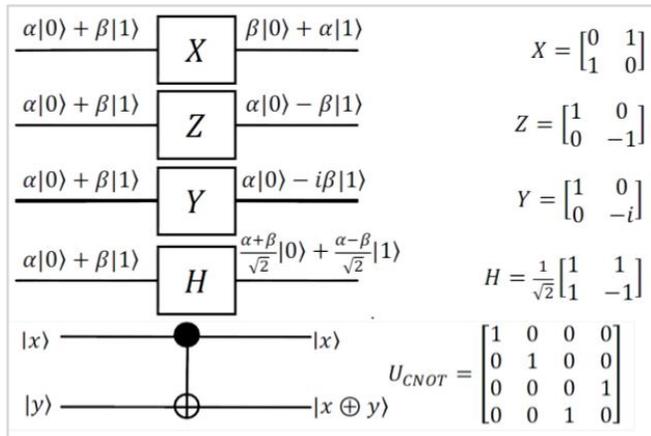
where $i = \sum_{j=0}^{n-1} 2^j i_j$.

Figure 1: Basic quantum gates with matrices presentation.

There are many quantum computation models can be used. Among them, quantum circuit model is related to mathematical model and it is suitable for simulation in **MATLAB** environment. In quantum circuit model, quantum information is presented by matrices and quantum operations are applied to qubit by quantum gates. Mathematically, we simulate the quantum gate via the matrix representation. Note that the condition for any quantum gate is revertible and the invert gate that move $\mathbf{U}|\psi\rangle$ back to $|\psi\rangle$ satisfy $\mathbf{U}^{-1} = \mathbf{U}^\dagger$, so $\mathbf{U}$ is unitary matrix. Fig. 1 shows the most important quantum gates with these matrices form. Pauli channel of quantum system consists of four basic elements, namely $\mathbf{X}$, $\mathbf{Z}$, $\mathbf{Y}$, and $\mathbf{I}$ (identity matrix). Any operation and errors acting on qubit can be represented as the combination of them. Hence, we have three types of errors which can effect on a qubit, they are bit flip, phase flip, and their combination. In general, the error operators that effect on n qubits have the form: $E = e_1 \otimes e_2 \otimes \ldots \otimes e_n$ where $e_i \in \{\mathbf{I}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}\}$.

3. THE PROPOSED SYSTEM MODEL AND SIMULATION OF BASIC QUANTUM CODES

A. Overview system

MATLAB program is a computing environment that based on the operation with matrices. Hence, it is useful tool in simulating the matrix formalism of quantum process. Since one qubit can be modelled by mathematical form by matrices of 2-dimensional Hilbert space

$$|\Psi\rangle = \begin{bmatrix} a \\ b \end{bmatrix}. \qquad (3)$$

the general quantum state of quantum system of n qubits can be given by n times tensor product via *kron* function of **MATLAB** program. In addition, any circuit is given as the series of unitary transformation applied to initial quantum state. These transformations can be represented by matrices form. As calculating by true table, we proposed two new matrices from original CNOT gate via **Fig. 2** and **Fig. 3**.
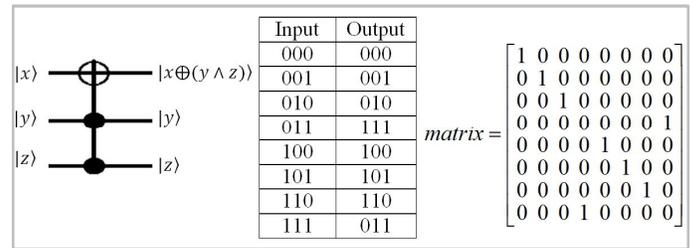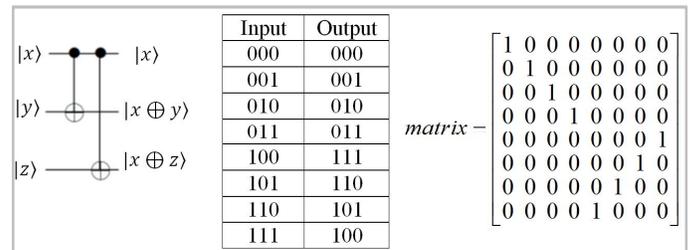

Figure 2: Type 1 modified CNOT gate.

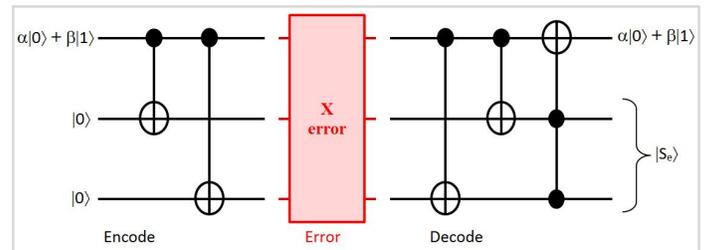**Figure 3:** Type 2 modified CNOT gate.

Figure 4: Quantum circuit for **X**-error 3- repetition code.
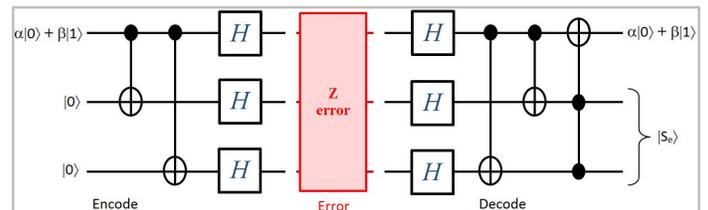

Figure 5: Quantum circuit for **Z**-error 3- repetition code.

B. Three Qubits Quantum Repetition Code

The simplest and the QECC is three qubits repetition code that can correct only one type error, namely bit flip or phase flip. The QECC can be simulated by quantum circuit model; they are showing in the **Fig. 4**, 5 for three qubits repetition code. The difference between those two quantum circuits based on the one type of Clifford gates: **X=HZH** and **Z=HXH**.

The MATLAB programs of quantum circuits in **Fig. 4**, 5 are described as follows. First, we create the initial message of one qubit to be protected. The quantum circuit starts with the initial information, we extend it to the 3-qubits system via helps of ancilla 2 qubits of zeros, after transformation by encode step, the logical states or encoded qubits are created. Here, two basis states of codewords for 3-qubit are defined as follows,

$$|\mathbf{0}_L\rangle = |000\rangle, |\mathbf{1}_L\rangle = |111\rangle. \qquad (4)$$

Then, the quantum gates as previous mentioned must to be declared. Here, the quantum gates **X**, **Z**, and two new types of CNOT gates are used. The *kron* function was used to extend one qubit to many qubits system.

C. Shor code

To extend the first full quantum code, Shor code for 9 qubits is created by Shor, which use both bit-flip correction and phase-flip correction and can correct bit-flip, phase-flip, and their combination. To do so, for one qubit is protected against phase-flip we need extend it to codeword of three qubits. Then, each qubits of that three-qubits need to extend to three-qubits to protect against bit-flip error. Hence, the quantum circuit starts with the initial information, we extend it to the 9-

qubits via helps of ancilla 8 qubits of zeros, after transformation by encode step, the logical states or encoded

qubits are created. Here, two basis states of codewords 9-qubits repetition as well as

$$|\mathbf{0}_L\rangle = \frac{1}{\sqrt{8}}(|000\rangle + |111\rangle)\otimes(|000\rangle + |111\rangle)\otimes(|000\rangle + |111\rangle),$$

$$|\mathbf{1}_L\rangle = \frac{1}{\sqrt{8}}(|000\rangle - |111\rangle)\otimes(|000\rangle - |111\rangle)\otimes(|000\rangle - |111\rangle).$$

Using matrices transformation, the states after applying error and decoding can be found. The final states show us the correction state can be recovered the syndrome $|S_e\rangle$ tell us which error has applied to logical states. The full quantum circuit for Shor code is given in **Fig.** 7. And the **MATLAB** program for Shor code is described in **Fig.** 8.

```
%Creat GATE:
%gate2CN=[ ---];
%gate2CT=[ ---];

%Creat ERRORS:
Xgate=[ 0 1;
       1 0];
Error0=eye(2^3);
Error1=kron(Xgate,eye(2));
Error1=kron(Error1,eye(2));
Error2=kron(eye(2),Xgate);
Error2=kron(Error2,eye(2));
Error3=kron(eye(2),eye(2));
Error3=kron(Error3,Xgate);
%----------------------
%Setup qubit input:
message=[3/5;4/5];
q_zero=qubit0(1);
q_one=qubit1(1);
q1=message(1,1)*q_zero+message(2,1)*q_one;
q2=qubit0(2);
%----------------------
%Init
Phi1=kron(q1,q2);
Phi2=gate2CN*Phi1;
%Applying Error
Phi3=Error3*Phi2;
%Decode
Phi4=gate2CN*Phi3;
Phi5=gate2CT*Phi4;
%------------
final=[];
final=[final;Phi5(i,n)];
fprintf('Finished!\n');
```

```
%Creat GATE:
%gate2CN=[ ---];
%gate2CT=[ ---];
H=hadamard(3);
%Creat ERRORS:
Zgate=[ 1 0;
        0 -1];
Error0=eye(2^3);
Error1=kron(Zgate,eye(2));
Error1=kron(Error1,eye(2));
Error2=kron(eye(2),Zgate);
Error2=kron(Error2,eye(2));
Error3=kron(eye(2),eye(2));
Error3=kron(Error3,Zgate);
%----------------------
%Setup qubit input:
q_zero=qubit0(1);
q_one=qubit1(1);
q1=(3/5)*q_zero+(4/5)*q_one;
q2=qubit0(2);
%----------------------
%Init
Phi1=kron(q1,q2);
Phi2=gate2CN*Phi1;
Phi3=H*Phi2;
%Applying Error
Phi4=Error3*Phi3;
%-------------
Phi5=H*Phi4;
%Decode
Phi6=gate2CN*Phi5;
Phi7=gate2CT*Phi6;
%-------------
final=[];
final=[final;Phi7(i,n)];
fprintf('Finished!\n');
```
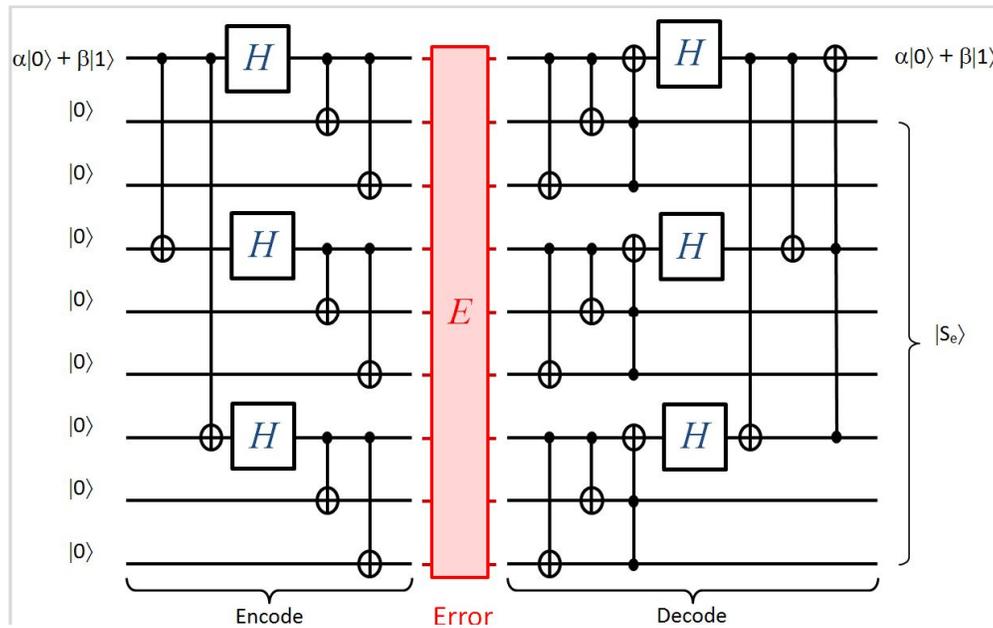
Figure 6: MATLAB program for quantum 3- repetition code.

Figure 7: Quantum circuit for Shor code.

```
%Creat GATE:                                              %Setup qubit input:
%gate2CN=[ ---];gate2CT=[ ---];                           q_zero=qubit0(1);
%-----------------------                                  q_one=qubit1(1);
gate1=kron(gateCNE(4),identify(2^5));                     q1=(3/5)*q_zero+(4/5)*q_one;
gate2=kron(gateCNE(7),identify(2^2));                     q2=qubit0(8);
H=hadamard(1);tem1=kron(H,identify(2^2));                 %-----------------------
tem2=kron(tem1,tem1);gate3=kron(tem2,tem1);               %Init
tem1=kron(gate2CN,gate2CN);gate4=kron(tem1,gate2CN);      Phi1=kron(q1,q2);
gate5=gate4;tem=kron(gate2CT,gate2CT);                    Phi2=gate1*Phi1;
gate6=kron(tem,gate2CT);gate7=gate3;gate8=gate2;          Phi3=gate2*Phi2;
gate9=gate1;gate10=kron(gate2CTE,eye(2^2));               Phi4=gate3*Phi3;
%-----------------------                                  Phi5=gate4*Phi4;
%Creat ERRORS:                                            %Applying Error
Ygate=[0 -i;                                              Phi6=Error0*Phi5;
       i 0];                                              %-----------------------
Error0=eye(2^9);                                          Phi7=gate5*Phi6;
Error1=kron(-i*Ygate,eye(2^8));tem=kron(eye(2^1),-i*Ygate); Phi8=gate6*Phi7;
Error2=kron(tem,eye(2^7));tem=kron(eye(2^2),-i*Ygate);    Phi9=gate7*Phi8;
Error3=kron(tem,eye(2^6));tem=kron(eye(2^3),-i*Ygate);    Phi10=gate8*Phi9;
Error4=kron(tem,eye(2^5));tem=kron(eye(2^4),-i*Ygate);    Phi11=gate9*Phi10;
Error5=kron(tem,eye(2^4));tem=kron(eye(2^5),-i*Ygate);    Phi12=gate10*Phi11;
Error6=kron(tem,eye(2^3)); tem=kron(eye(2^6),-i*Ygate);   %-----------------------
Error7=kron(tem,eye(2^2));tem=kron(eye(2^7),-i*Ygate);    final=[];
Error8=kron(tem,eye(2^1));                                final=[final;Phi12(i,n)];
Error9=kron(eye(2^8),-i*Ygate);                           fprintf('Finished!\n');
```

Figure 8:MATLAB program for Shor code.

4. CONCLUSION

The paper presents basic information on quantum error correction, using MATLAB environment the three-qubits repetition and nine-qubits Shor code have been proved to correct. Such simulation of simplest QECC help us better understanding of design and quantum error correction and quantum mechanism for further.

The outstanding result prove that the proposed framework is novel for further researches simulation of quantum information system. In the future, we plan to use this framework for simulation of quantum stabilizer codes. The application of quantum stabilizer codes on quantum algorithm, quantum communication can be achieved by proposed protocol.

Conflict of Interest: On behalf of all authors, the corresponding author declares that there is no conflict of interest.

REFERENCES

1. Bennett, C.H., Shor, P.W. **Quantum information theory**. (1998). IEEE Transaction on Information theory. 44(6):2724-2742.
   https://doi.org/10.1109/18.720553
2. Shor, P.W. **Algorithms for quantum computation: discrete logarithms and factoring.** (1994). Proceedings 35th Annual Symposium on Foundations of Computer Science. IEEE Comput. Soc. Press.
3. Grover, L.K. **Quantum Mechanics Helps in Searching for a Needle in a Haystack.** (1997). Phys. Rev. Lett. 79:325.
4. Nguyen, D.M., Kim, S. **Quantum Key Distribution Protocol Based on Modified Generalization of Deutsch-Jozsa Algorithm in d-level Quantum System.** (2019). Int. J. Theor. Phys. 58(1):71-82.
   https://doi.org/10.1007/s10773-018-3910-4
5. Nguyen, D.M., Kim, S. **Multi-Bits Transfer Based on the Quantum Three-Stage Protocol with Quantum Error Correction Codes**. (2019). Int. J. Theor. Phys. 58(6):2043-2053.
   https://doi.org/10.1007/s10773-019-04098-4
6. Nguyen, D.M., Kim, S. **The fog on Generalized teleportation by means of discrete-time quantum walks on N-lines and N-cycles.** (2019). Modern Physics Letters B. 33(23):1950270.
   https://doi.org/10.1142/S0217984919502701
7. Peter W. Shor, **Scheme for reducing decoherence in quantum memory**. (1995). Phys. Rev. A. 52(4).
   https://doi.org/10.1103/PhysRevA.52.R2493
8. Nguyen, D.M., Kim, S. **Quantum stabilizer codes construction from Hermitian self-orthogonal codes over GF(4)**. (2018). Journal of Communications and Networks. 20(3):309-315.
9. Nguyen, D.M., Kim, S. **New Constructions of Quantum Stabilizer Codes Based on Difference Sets.** (2017). Symmetry. 10(11):655.
   https://doi.org/10.3390/sym10110655
10. Nguyen, D.M., Kim, S. **New construction of binary and nonbinary quantum stabilizer codes based on symmetric matrices**. (2019). International Journal of Modern Physics B. 33(24):1950274.
11. Nguyen, D.M., Kim, S. **Construction and complement circuit of a quantum stabilizer code with length 7**. (2016). Eighth International Conference on Ubiquitous and Future Networks (ICUFN). 332 - 336.
12. Noson.S. Yanofsky, Micro.A. Mannucci, **Quantum computing for computer scientists**. (2008). Cambridge University Press New York, NY, USA.
13. Zidan, M., et. al. **A Novel Algorithm based on Entanglement Measurement for Improving Speed of Quantum Algorithms.** (2018). Appl. Math. Inf. Sci. 12(1):265-269.
    https://doi.org/10.18576/amis/120127
14. Zidan, M., et. al. **Quantum Classification Algorithm Based on Competitive Learning Neural Network and Entanglement Measure.** (2019). Appl. Sci. 9(7):1277.
15. Zidan, M., et. al. **A quantum algorithm based on entanglement measure for classifying Boolean multivariate function into novel hidden classes.** (2019). Results in Phys. 15:102549.
    https://doi.org/10.1016/j.rinp.2019.102549
16. K. Ravi, N. K. Pandy, A. S. Kumar, and N. Kushal. **Quantum Computer-Hardware**, International Journal of Advanced Trends in Computer Science and Engineering, Vol. 5, No. 4, pp. 46-53, 2016.
17. K. Ravi, N. K. Pandy, A. S. Kumar, and N. Kushal. **Quantum Computer-Algorithms**, International Journal of Advanced Trends in Computer Science and Engineering, Vol. 5, No. 4, pp. 54-60, 2016.
18. A. Naincy, B. Pratap. **Develop a Hybrid Method to Encode Data**, International Journal of Advanced Trends in Computer Science and Engineering, Vol. 6, No. 3, pp.51-56, 2017.
19. Nguyen, D.M., Kim, S. **A novel construction for quantum stabilizer codes based on binary formalism**. (2020). International Journal of Modern Physics B. 34(8):2050059.
    https://doi.org/10.1142/S0217979220500599
20. Nguyen, D.M., Kim, S. **Quantum stabilizer codes based on a new construction of self-orthogonal trace-inner product codes over GF(4)**. (2020). International Journal of Modern Physics B. 34(5):2050017.
    https://doi.org/10.1142/S0217979220500174