

A Comparative Study On Image Deblurring Techniques



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Abstract Image blur is a common problem that occurs when recording digital images due to camera shake, long exposure time, or movement of objects. As a result, the recorded image is degraded and the recorded scene becomes unreadable. Recently, the field of blur removal has gained increasing interest in a lot of researches. The problem is known as blind deconvolution if the only available information is the blurred image and there is no knowledge about the blurring model or the Point Spread Function (PSF). In this case, the basic target of the process is to recover both the blur kernel and the deblurred (latent) image, simultaneously. In this paper, we introduced a comprehensive study on the image deblurring, type of blur, noise model and finally a comparative study of different image deblurring techniques. We performed several experiments to evaluate these techniques in terms of performance, blur type, Peak Signal to Noise Ratio and structural similarity (SSIM).

Key words : Image Deblurring, blur, PSF, degradation model, image enhancement and restoration, PSNR.

INTRODUCTION

Images are widely used in many kinds of applications such as everyday photography, monitoring, medical imaging, astronomy, microscopy, and remote sensing. Digital images are composed of picture elements or pixels that are organized in a grid. Each pixel contains an intensity value which determines the tone at a specific point. Unfortunately, all captured images end up more or less blurry. The motion of objects or the vibration of the sensor (camera) when pressing the shutter causes the image to be blurred. There are many factors that cause the blurring or degradation of the digital image, such as movement during the capture process, using long exposure times, using wide angle lens, etc.[2]. However, there are two main causes for motion blur: (i) the image is blurred by the camera vibration which causes all pixels in the image to be affected, and (ii) the image is blurred by object motion which causes a specific region to be blurred. Image blur usually devastate the images, and practically it is hard to avoid it because there is a lot of interference in the environment. Image deblurring is the process of applying and solving mathematical models to recover the original (sharp) image. Recently, image deblurring and restoration become the main and the most important sub field of digital image processing. Image deblurring is used to make images sharp and useful. There are many applications for image deblurring such as:

- Recovering valuable photographs.
- Watching distant star fields through ground based telescopes.

- Watching space vehicles and satellites.
- Radar imaging
- Tomography and medical imaging
- Microscopy
- Iris recognition

Mathematically, the blurring can be defined as a linear process that leads to image degradation. The blurring model can be described as a combination of two operations; the first operation is convolving the original (unblurred or sharp) image with unknown kernel; the second operation is the addition of some noise to the resulting image. Equation 1 introduces the notations that are used in the blurring process[1].

$$B = I \otimes K + N \quad (\text{Eq. 1})$$

Where B is blurred image, I is the unblurred image, K is unknown linear shift-invariant point spread function (PSF) or impulse response, and N is the additive noise. For example, if the original (unblurred) image contains a single bright (white) pixel and all other pixels are black. When capturing this image, the single bright pixel is spread over its neighboring pixels. This single bright pixel is called a point source. The point spread function (PSF) is a function that describes the response of an imaging system to a point source. Point Spread Function (PSF) is the degree to which an optical system blurs (spreads) a point of light[26]. Optical Transfer Function (OTF) is the Fourier transfer of the point Spread Function (PSF) and the PSF is the inverse Fourier transform of OTF. In the frequency domain, the OTF describes the response of a linear, position-invariant system to an impulse[27]. Image deblurring can be viewed as a process that tries to obtain an approximation of I. One possible solution to obtain the unblurred image is the blind deconvolution which requires the estimation of K using image intensities and gradients to deconvolve B. Equation 1 contains a term for the noise that are consequences of problems associated with acquisition devices.

In the literature, several image restoration techniques have been proposed, some of these approaches are non-blind or they require a priori knowledge to estimate the blur filter(PSF), and the others are blind which means that they can perform image Deblurring without a priori knowledge of the blur filter(PSF). In this paper we study different approaches and methodologies for image deblurring.

BLURRING

It is well known that an image is clear if we can perceive the shape of all its objects correctly. For example, a face image is clear if we can recognize lips, eyes, nose, etc. The shapes of contained objects are recognized from its edges. By contrast, blurring means reducing the edge content and making the transition from one color to another very smooth. One may

think that zooming is one type of blurring. Actually, if an image is zoomed using pixel replication and large zooming factor, the resulting image is a blurred image. however, the difference between zooming and blurring is that the number of pixels in the zoomed image is larger than the number of pixels in the original image, but the number of pixels in the original image and blurred one is the same. Blurring can be thought of as applying some filter to an image. There are many kind of filters that can be used to perform blurring, such as average filter, weighted average filter, Gaussian filter and motion filter. The average filter has three properties: it is odd ordered, the sum of all its elements equal 1, and all its elements are the same. The amount of the blurring is increased when the size of the kernel is increased. That is because a large number of pixels are included and one smooth transition is defined. The weighted average filter gives more weight to the center value. As a result, the contribution of the center pixel becomes more than the rest of the pixels. The Gaussian blur uses a Gaussian function to calculate the transformation that will be applied to each pixel in the image. It is low-bass pass filter which is always used to reduce noise and enhance image structure at different scales. In Gaussian blur, the pixel weights are decreased from kernel centre to edges according to a bell-shaped curve. The Gaussian filter blends a specific number of pixels incrementally, following a bell-shaped curve. The blurring is dense in the centre and feathers at the edge[3]. It can be used to give more control over the Blur effect because the Gaussian blur depends on the Size and Alfa. Motion blur is the apparent streaking of rapidly moving objects in a still image[5]. It occurs when the image being recorded is changed during the recording either due to rapid movement or long exposure. The basic reason for all motion blur is the relative motion between the recording device and the scene in the form of translation, rotation, sudden change of scale, or a combinations of them [2]. The Motion Blur is a filter that adds a blur in a specific direction, so the image appears to be moving. The motion is controlled by an angle (0 to 360 degree) or direction (-90 to 90) and by distance or intensity in pixels. There are many other types of blur types such as out-of-focus blur. If a camera is used to convert a 3D real scene into a 2D image, some parts of the scene are in focus while other parts are not. If the aperture of the camera is circular, the image of any point source is a small disk, known as the circle of confusion (COC)[6]. The degree of defocus (diameter of the COC) depends on several factors including: the focal length, the aperture number of the lens, and the distance between the camera and the scene.

DEBLURRING TECHNIQUES

GENERAL LINEAR MODEL

In this model[7], it is assumed that the blurring process (the process of converting sharp image into blurred image) is linear. This assumption is very useful because in many cases the blur can be well approximated by a linear model. It is also assumed that the desired sharp image I and the recorded blurred image B are two grayscale digital images of size $m \times n$. The most simple case of blurring occurs when the blurring of the rows is performed independently of the blurring of the columns. In this case, there are two matrices A_c and A_r of sizes $m \times m$ and $n \times n$, respectively. The

relation between the blurred and sharp images is expressed as:

$$B = A_c I A_r^T \quad (\text{Eq. 2})$$

The term $A_c I$ denotes applying the same vertical blurring operation to all columns of I . The term $I A_r^T$ denotes applying the same horizontal blurring to all rows of I . The very naïve solution to this linear model is:

$$I = A_c^{-1} B (A_r^T)^{-1} \quad (\text{Eq. 3})$$

Unfortunately, the reconstructed image does not resemble the desired image because we have omitted the noise term. Image noise is a random variation of intensities (color values) in the image and is usually generated by the sensor and the circuitry of the camera. Image noise introduces a set of spurious and extraneous information, so it should be considered in the linear blurring model as expressed in equation 4.

$$B = A_c I A_r^T + E \quad (\text{Eq. 4})$$

Where the noise image E is also of size $m \times n$. So the sharp image can be reconstructed using equation 5.

$$I = A_c^{-1} B (A_r^T)^{-1} - A_c^{-1} E (A_r^T)^{-1} \quad (\text{Eq. 5})$$

The term $A_c^{-1} E (A_r^T)^{-1}$ is informally called the inverted noise which measures the contribution of the additive noise in the reconstruction of the sharp image. The more the value of this term, the more the inverted noise dominates the reconstruction process. Practically, It is noticed that the inverted noise always dominates the reconstruction process.

SINGULAR VALUE DECOMPOSITION

In this section, we will consider a blurring model [7] that is more general than the previous one. Here, the blurring is performed on the rows and columns of the image simultaneously. The first step is to reshape the matrices I and B to be two column vectors of length $N = mn$ by stacking the columns of these matrices into two long column vectors I_{vec} and B_{vec} . The general linear blurred model can be described as a relation between the sharp column vector I_{vec} and blurred column vector B_{vec} as expressed in equation 6.

$$B_{vec} = A I_{vec} + e \quad (\text{Eq. 6})$$

At this point, we will assume that A is a known blurring matrix of size $N \times N$, but in practice, it is constructed from the imaging system. The term e denotes the column vector of the noise image E . So, the sharp image can be reconstructed by solving equation 7.

$$I_{vec} = A^{-1} B_{vec} - A^{-1} e \quad (\text{Eq. 7})$$

Where the term $A^{-1} e$ is the inverted noise. If the deblurred image is very distorted, it means that the inverted noise term dominates the reconstructed image. The blurring matrix A can be decomposed into two orthogonal matrices U and V , and a diagonal matrix Σ using the singular value decomposition (SVD) as expressed in equation 8.

$$A = U \Sigma V^T \quad (\text{Eq. 8})$$

Where $U^T U = V^T V = I_N$ and $\Sigma = \text{diag}(\sigma_i)$ is $N \times N$ diagonal matrix which contains a set of nonnegative values (Singular values) that are organized in the main diagonal in a nonincreasing order, i.e. $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_N \geq 0$. The number of positive singular values determines the rank of A . The columns of U are called the left singular vectors. The columns of V are called the right singular vectors. When i is increased, the singular values decreased and the corresponding singular vectors tend to have more sign changes to represent higher-frequency information. If all singular values are not zeros, we can directly deduce that the

inverse of the blurring matrix A can be expressed as shown in equation 9.

$$A^{-1} = V \Sigma^{-1} U^T \quad (\text{Eq. 9})$$

Where Σ^{-1} is also a diagonal matrix whose elements in the main diagonal are $1/\sigma_i$, for $i = 1, 2, \dots, N$.

Equation 8 and 9 can be reformulated in terms of the left singular vectors u_i , right singular vectors v_i and singular values σ_i as shown in equation 10 and 11.

$$A = \sum_{i=1}^N \sigma_i u_i v_i^T \quad (\text{Eq. 10})$$

$$A^{-1} = \sum_{i=1}^N \frac{1}{\sigma_i} v_i u_i^T \quad (\text{Eq. 11})$$

So, equation 7 can be rewritten as shown in equation 12.

$$I_{vec} = A^{-1} B_{vec} - A^{-1} e \quad (\text{Eq. 12})$$

$$I_{vec} = \sum_{i=1}^N \frac{u_i^T B_{vec} v_i}{\sigma_i} - \sum_{i=1}^N \frac{u_i^T e v_i}{\sigma_i} \quad (\text{Eq. 13})$$

When the percentage of the first singular value to the last singular value, i.e. σ_1/σ_N , is very large, it means that the solution is highly sensitive to perturbations and rounding errors.

RICHARDSON-LUCY DECONVOLUTION APPROACH

The Lucy-Richardson algorithm, or Richardson-Lucy deconvolution, is an iterative procedure for recovering the sharp image that has been blurred by a known point spread function (PSF). It was developed by William Richardson and Leon Lucy. In this technique, the Pixels in the observed (blurred) image is represented in terms of the point spread function and the latent (sharp) image. Some of the researches that used Lucy-Richardson algorithm can be found in [2, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. Let K_{ij} is the PSF (the portion of light that comes from true location j and is observed at another location i), x_j is the pixel value at location j in the latent (sharp) image, and b_i is the observed value at pixel location i . The pixels in the observed image are represented in terms of the point spread function and the latent (sharp) image as expressed in equation 14.

$$b_i = \sum_j K_{ij} x_j \quad (\text{Eq. 14})$$

The model assumes that x_j values are Poisson distributed, which is the most suitable for photon noise in the data. The model calculates the most likely x_j given the observed b_i and known PSF K_{ij} . The model finds a recursive formula to estimate x_j as expressed in equation 15.

$$x_j^{(t+1)} = x_j^{(t)} \sum_i \frac{b_i}{c_i} K_{ij} \quad (\text{Eq. 15})$$

$$c_i = \sum_j K_{ij} x_j^{(t)} \quad (\text{Eq. 16})$$

Experimental results showed that the iteration converges to the most likelihood solution of x_j .

NEURAL NETWORK APPROACH

In this approach, neural network are employed to convert blurred images into sharp images. The basic idea is that the system can be trained on various types of blurred images and their corresponding sharp ones. The input to the training

system is a list of blurred images and the target output is the corresponding sharp images. After the system is trained on various types of blurred images, it becomes able to predict an unknown sharp image from the blurred one. Some of the techniques that employed neural network, in order to converge the recorded blurred image to the sharp version of it, can be found in [32, 33, 34, 35, 37]. Jubien et al. [31] described two different training algorithms that can be used by the neural network for blind image restoration, one is based on the least mean squares (LMS) rule, and the second is called dubbed algorithm-X. Subashini et al. [29] adopted back propagation network with gradient decent rule which consists of three layers. They used highly nonlinear back propagation neuron for image restoration to get a high quality restored image and achieves fast neural computation, less computational complexity due to the less number of neurons used and quick convergence without lengthy training algorithm. S. Saadi et al. [30] proposed improving the training of the neural network using a novel swarm optimization algorithm called Artificial Bees Colony (ABC), inspired from the foraging intelligence of honey bees. Z. Hongying et al. [36] proposed a variational image deblurring algorithm based on modified Hopfield neural network (MHNN).

ITERATIVE METHODS

J. Biemond [75] discussed using iterative restoration techniques for the removal of linear blurs from images that are degraded by pointwise nonlinearities such as film saturation and additive noise. They show that iterative algorithms can use various types of prior knowledge about the class of possible solutions to remove non-stationary blurs. They presented study on the convergence problem of the algorithms. They also compared between classical solutions such as inverse filters, Wiener filters, and constrained least-squares filters which are shown to be limiting solutions of variations of the iterations. Finally, they introduced Regularization as a means for preventing the excessive noise magnification that is typically associated with ill-conditioned inverse problems such as the deblurring problem, and they showed that noise effects can be minimized by terminating the algorithms after a finite number of iterations.

A. Beck and M. Teboulle [76] studied a class of Iterative Shrinkage-Thresholding Algorithms (ISTA) for solving linear inverse problems arising in image processing. They reported that these methods is attractive due to its simplicity but they are also known to converge very slowly. Then, they presented a Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) which preserves the computational simplicity of ISTA, but with a global rate of convergence which is proven to be significantly better, both theoretically and practically.

James L. Herring [77] showed the effectiveness of three projected iterative methods for image deconvolution: projected successive over-relaxation method (SOR), projected Landweber method, and an interior point gradient method. J. G. Nagy et al. [78] introduced a set of Matlabtools that implement iterative image restoration methods. They uses Matlab for the implementation of efficient matrix vector multiplication, and for solving the linear system for preconditioners, efficiently. They also combine the powerful scientific computing and graphics capabilities in Matlab to do object-oriented programming and operator overloading, results in a set of usable and extensible classes.

WIENER FILTERING

One of the most common technique for image deblurring is wiener filtering. The wiener filter has a large ability to remove the blur in images caused by linear motion or unfocussed optics. The blurred image can be seen as a result of poor sampling. Each pixel in the image should contain intensity value for a single stationary point in front of the capturing device(or camera). Unfortunately, if the camera is moved or the shutter speed is very slow, a given pixel will be an amalgam of intensities from points along the line of the camera's motion[38, 39]. The Wiener filter performs an optimal trade-off between inverse filtering and noise smoothing because it removes the additive noise and inverts the blurring simultaneously. Some of the techniques that used Wiener filtering can be found on [40, 41, 42, 43, 44, 45, 46, 47]. The model works as described in equations 17-20. Let the case is the model described in equation 17.

$$b(x, y) = h(x, y) * s(x, y) + n(x, y) \quad (\text{Eq. 17})$$

Where $s(x, y)$ is the unknown sharp image, $h(x, y)$ is the known impulse response of a linear scale-invariant system, $n(x, y)$ is some unknown additive noise independent of $s(x, y)$, and $b(x, y)$ is the observed blurred image. The Wiener deconvolution filter finds $g(x, y)$ and use it to estimate $s(x, y)$, as expressed in equation 18.

$$\hat{s}(x, y) = g(x, y) * b(x, y) \quad (\text{Eq. 18})$$

Where $\hat{S}(x, y)$ is an estimate of $s(x, y)$ that minimizes the mean square error. The filter works in the frequency domain. The first step is to calculate the frequency domain version of $g(x, y)$, as expressed in equation 19.

$$G(u, v) = \frac{H(u, v) W(u, v)}{|H(u, v)|^2 W(u, v) + N(u, v)} \quad (\text{Eq. 19})$$

Where $G(u, v)$ is the Fourier transform of g , $H(u, v)$ is the Fourier transform of h , $W(u, v)$ is the mean power spectral density of $s(x, y)$, $N(u, v)$ is the mean power spectral density of the noise $n(x, y)$. Finally, the filtering is performed in the frequency domain, as expressed in equation 20.

$$\hat{S}(u, v) = G(u, v)B(u, v) \quad (\text{Eq. 20})$$

Where $\hat{S}(u, v)$ is the Fourier transform of the estimated sharp image and $B(u, v)$ is the Fourier transform of the observed blurred image. If three pixels in a line contain info from the same point on an image, the digital image will seem to have been convolved with a three-point boxcar in the time domain[39]. It is seen that the technique is based on inverse filtering. Unfortunately, there is a number of drawbacks associated with the wiener filter: (i) H is unknown. It can be guessed for a given image but it requires a lot of trials and efforts to give a good estimation of H ; (ii) it is failed in some cases because the sine function equal 0 at some values of x and y .

BLIND DECONVOLUTION APPROACH

It is noticeable that inverse filtering and wiener filtering require accurate estimation of the degradation function and also require information about the noise model for image restoration. But it is very hard to obtain from a practical point of view. So, blind deconvolution approach is not concerned with an accurate estimation of the degradation function. Instead, an initial estimation of the degradation function is calculated. Then the system carries out an iterative process of blind deconvolution until an accurate estimation is obtained which minimizes the mean square error. Some of the

researches that discusses blind deconvolution approach can be found in [2, 10, 28, 48, 49, 51]. There are two basic approaches for blind deconvolution. The first is called projection based blind deconvolution and the second is the maximum likelihood restoration. The first approach makes an initial estimation of the point spread function (PSF) followed by an initial estimation of the true image. Then, the process is repeated until a predefined convergence criterion is met. The advantages of this approach is that it is robust against inaccuracies of support size and it is insensitive to noise. The disadvantages of this approach is that it is not unique and can lead to errors if unsuitable local minima has been initiated. The second approach uses the maximum likelihood to estimate parameters such as PSF and covariance matrix. Since the estimation of PSF is not unique, the approach can consider parameters such as size, symmetry, and other parameters. The advantages of this approach is low computational complexity. The approach can obtain blur, noise and power spectra of the sharp image. The disadvantage of this approach is that it converges to local minima of the estimated cost function.

SPARSE REPRESENTATION

Most of the blind image deblurring techniques either only remove simple motion blurring, or need user interactions to handle more complex cases. Sparse decomposition is a process that estimates a sparse multi-dimensional vector that satisfies a set of equations in a linear system given high-dimensional observed data and a design matrix. It is used in a wide range of applications such as image deblurring and restoration. Sparse representation succeeds due to the development of l_1 -norm optimization techniques, and also because natural images are intrinsically sparse in some domain. Some of the techniques that use sparse approximation for blind image deblurring can be found on [55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69]. J. Yang et al. [54] presented an approach for image super-resolution using sparse signal representation. From image statistics, they stated that image patches can be well-represented as a sparse linear combination of elements from an appropriately chosen over-complete dictionary. So they seek a sparse representation for each patch of the low-resolution input, and then use the coefficients of this representation to generate the high-resolution output. W. Dong et al. [52] introduced two adaptive regularization terms into the sparse representation framework. The first term is a set of autoregressive (AR) models that are learned from a specific dataset. The best fitted AR model is adaptively selected to regularize the image local structures. The second term is the image non-local self-similarity. Experimental results on image deblurring and super-resolution shows that the usage of adaptive sparse domain selection and adaptive regularization, achieves an improved results in terms of PSNR and visual perception. J.-F. Cai et al. [53] presented an approach to remove motion blurring from a single image by formulating the blind blurring as a new joint optimization problem, which simultaneously maximizes the sparsity of the blur kernel and the sparsity of the clear image under certain suitable redundant tight frame systems (curvelet system for kernels and framelet system for images). The advantage of this techniques is that it can recover high-quality images from given blurred images without any prior knowledge of the blur kernel.

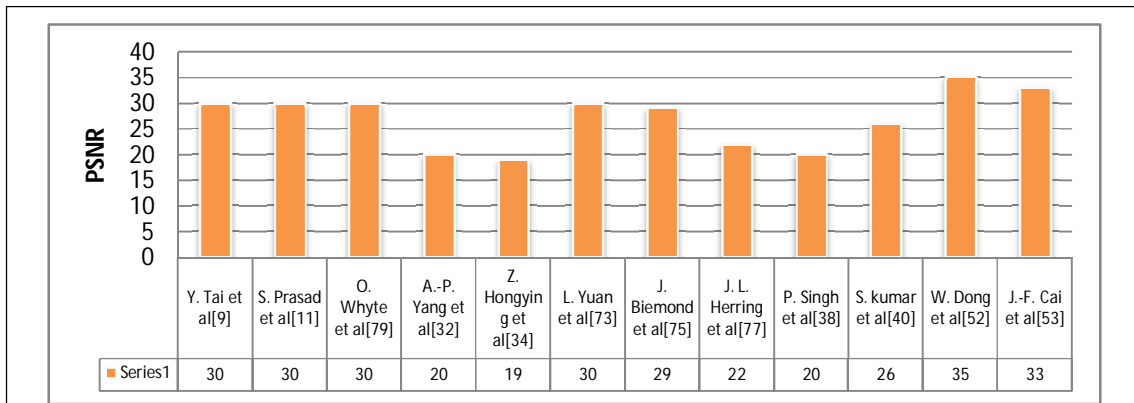


Figure 1: Mean PSNR (db) for each of the tested techniques

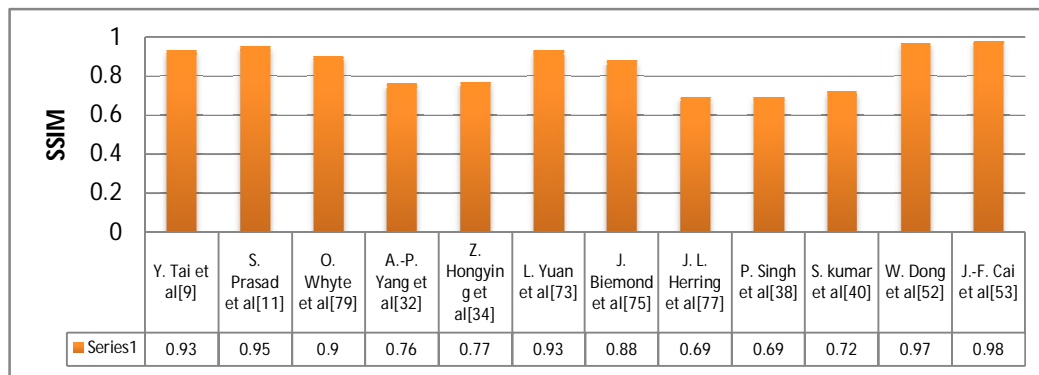


Figure 1: Mean SSIM for each of the tested techniques

OTHER APPROACHES

As we discussed before, non-blind deconvolution methods have been widely used for image deblurring. These techniques can handle only the linear blur model where the blurred image is generated by a linear convolution of the latent image and the blur kernel. However, this often fails because there are several types of outliers that should be handled. S. Cho et al. [70] proposed a novel blur model that explicitly takes into account several types of outliers, and build a robust non-blind deconvolution method that reduces the visual artifacts caused by outliers. They tested the proposed system on synthetic and real-world examples. It was shown that if these outliers are not handled, they cause severe ringing artifacts even if the kernel is estimated accurately. So they analyze a few common types of outliers that cause other deblurring techniques to fail, such as pixel saturation and non-Gaussian noise. H. Yin and I. Hussain[72] proposed a blind deconvolution method for image deblurring based on ICA measure as well as a simple genetic algorithm. S. El-Regaily, et al. [71] proposed an algorithm for image deblurring using Genetic Algorithms, by finding proper parameters and goal function.

O. Whyte et al. [79] propose a new parametrized geometric model of the blurring process in terms of the rotational motion of the camera during exposure. The model can capture non-uniform blur in an image due to camera shake using a single global descriptor, and can be substituted into existing deblurring algorithms with only small modifications. L Yuan et al. [73] proposed an approach that uses the blurred image and an assistance noisy image to estimate an accurate

blur kernel, which is harder to obtain from a single blurred image. Then, they used a residual deconvolution to significantly reduce ringing artifacts inherent to image deconvolution. Finally, they used a gain-controlled deconvolution process to suppress the remaining ringing artifacts in smooth image regions. A. Gupta et al. [74] presented an image deblurring technique to estimate spatially non-uniform blur. They use existing spatially invariant deconvolution methods to compute initial estimates of the latent image. They represented the camera motion as a Motion Density Function (MDF) which records the fraction of time spent in each discretised portion of the space of all possible camera poses. They reported that 6D camera motion is well approximated by 3 degrees of motion, then they analyse the scope of this approximation.

EXPERIMENTAL RESULTS

To evaluate the effectiveness of different image deblurring techniques, the quality of the latent image is measured. There are two types [50] of measures that can be used to evaluate the quality: subjective measures and objective measures. Subjective measures measure the user's satisfaction or perception without calculating a numeric quantity through an explicit formula. This type of metrics is the most accurate because it is directly trying to accommodate different user requirements. The second type of metrics is the objective metrics which use an explicit formula to calculate a numeric quantity. There are two types of objective metrics that are widely used in the evaluation of image deblurring techniques: Peak Signal-To-Noise Ratio (PSNR) and Structural Similarity (SSIM). PSNR is used to measure the quality of

image in dB (decibels). PSNR between two images (deblurred image and original image) measures how far the two images are equal.

$$MSE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} [I_{ij} - X_{ij}]^2 \quad (\text{Eq. 21})$$

$$PSNR = 10 \log_{10} \left(\frac{(MAX^2)}{MSE} \right) \quad (\text{Eq. 22})$$

Where I is the original image, X is the restored (deblurred) image, M is the number of rows, N is the number of columns and MAX is the maximum possible intensity value of the image. The Structural Similarity (SSIM) [50] is calculated as expressed in equation 23.

$$SSIM = \frac{(2\mu_I\mu_X + c_1)(2\sigma_{IX} + c_2)}{(\mu_I^2 + \mu_X^2 + c_1)(\sigma_I^2 + \sigma_X^2 + c_2)} \quad (\text{Eq. 23})$$

Where μ_I and μ_X are the means of I and X , respectively; σ_I^2 and σ_X^2 are the variance of I and X , respectively; c_1 and c_2 are used to stabilize the division with weak denominator.

Each of the listed techniques in figure 1 and figure 2 is tested on 20 different blurred images. We added a Gaussian noise to each blurred image. The standard deviation of the Gaussian noise is 0.005 to simulate the realistic conditions during image acquisition. The obtained PSNR and SSIM are recorded for each one of the tested technique.

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