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Fitting A Set Of Data Points To Fourier Series Expansions

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ABSTRACT

Usually we are given a set of data points to which we fit our Fourier coefficients against in order to get the best fit possible. Here in our technique We can use Fourier series for fitting a set of data up to a certain harmonic in a fast and precise way which given us the facility of dealing a certain Fourier expansion rather than the original data points.

Keywords
Fourier series, harmonic analysis, third harmonic

1. INTRODUCTION

The usual mathematical representation of an image is a function of two spatial variables $f(x, y)$. The value of the function at a particular location (x, y) represent the intensity of the image at that point. This is called the spatial domain. The term transform refers to an alternative mathematical representation of an image. For example the Fourier transform is a representation of an image as a sum of complex exponentials of varying magnitudes, frequencies, and phases.

Since Fourier's time, many different approaches to defining and understanding the concept of Fourier series have been discovered, all of which are consistent with one another, but each of which emphasizes different aspect of the topic. Some of the more powerful and elegant approaches are based on mathematical ideas and tools that were not available at the time Fourier completed his original work. Fourier originally defined the fourier series for real-valued functions of real arguments, and using the sine and cosine functions as the basics set for the decomposition.[1][2]

Fourier series has long providy of Fourier die one of the principle methods of analysis for mathematical physics, engineering, and signal processing it has spurred generalizations and applications that continue to develop right up to the present. While the original theory of Fourier series applies to periodic functions occurring in wave motion, such as with light and sound, its generalizations often relate to wider setting, such as the time-frequency

analysis underlying the recent theories of wavelet analysis and local trigonometric analysis.[3]

Many other Fourier related transforms have since been defined, extending the initial idea to other applications. This general area of inquiry is now sometimes called harmonic analysis. A Fourier series, however can be used only for periodic functions, or for functions on a bounded (compact) interval.

A Fourier series is an expansion of periodic function $f(x)$ in terms of infinite sum of since and cosines .Fourier series make use of the orthogonally relationships of the sine and cosine functions. The computation and study of Fourier series is known harmonic analysis and is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in , solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical .[4]

2. HISTORICAL BACKGROUND

There are antecedents to the notion of Fourier series in the work Euler and D.Bernoulli on vibrating string, but the theory of Fourier series truly began with the profound work of Fourier on heat condition at the beginning of the 19^{th} century. In [5], Fourier deals with the problem of describing the evolution of the temperature $T(x,t)$ of a thin wire of the length π, stretched between $x = 0$ and $x = \pi$,with a constant zero temperature at the ends: $T(x,t) = 0$ and $T(\pi,t) = 0$. He proposed that the initial temperature $T(x,0) = f(x)$ could be expanded in a series of sine functions:

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \qquad (1)$$

with

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx dx \qquad (2)$$

Although Fourier did not give a convincing proof of convergence of the infinite series in eq.(1), he did offer the conjecture that convergence holds for an "arbitrary" function f . In addition to positing (1)and(2), Fourier argued that the temperature $T(x,t)$ is a solution to the following heat equation with boundary condition:

$$\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial x^2},\ 0<x<\pi, t>0$$

$$T(0,t)=T(\pi,t)=0, \quad t\geq 0$$

$$T(x,0)=f(x), \quad 0\leq x\leq \pi$$

Making use of (1), Fourier showed that the solution $T(x,t)$ satisfies

$$T(x,t)=\sum_{n=1}^{\infty} b_n e^{-n^2 t}\sin nx \quad (3)$$

This was the fist example of the use of Fourier series to solve boundary value problem in pattial differential equations. To obtain (3), Fourier made use of D.Bernoulli`s method of separation of variables ,which is now a standard technique for solving boundary value problem.

3. DEFINITION OF FOURIER SERIES

The Fourier sine series, defined in eq.s(1)and(2), is a special case of a more general concept: the Fourier series for a periodic function. Periodic functions arise the study of the wave motion, when a basic waveform repeats it self periodically. Such periodic wave forms occur in musical tones, in the plane waved of electromagnetic vibrations, and in the vibration of strings. These are just a few examples. Periodic effects also arise in the motion of the planets, in ac-electricity, and(to a degree) in animal heartbeats.[5],[9]

A function f is said to have period P if $f(x+P)=f(x)$ for all x. For notational simplicity, we shall restrict our discussion to functions of period 2π. There is no loss of generality in doing so, since we can always use a simple change of scale $x=(P/2\pi)t$ to convert a function pf period P into one of period 2π.

If the function f has period 2π ,then its Fourier series is

$$c_0+\sum_{n=1}^{\infty}\{a_n\cos nx+b_n\sin nx\} \quad (4)$$

With fourier coefficients $c_0, a_n and b_n$ defined by the integrals

$$c_0=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx \quad (5)$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx, \quad (6)$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx. \quad (7)$$

The sine series defined by (1) and (2) is a special instance of Fourier series if f is initially defined over the interval $[0,\pi]$, then it can be extended to $[-\pi,\pi]$(as an odd function)by letting $f(-x)=f(x)$, and then extended periodically with period $p=2\pi$

The Fourier series for this odd, periodic function reduces to the sine series in eq.s(1) and (2),because $c_0=0$, each $a_n=0$, and each b_n in eq.(7) is equal to the b_n in eq.(2)

it is more common nowadays to express Fourier series in an algebraically simpler form involving complex exponentials. Following Euler, we use the fact that the complex exponential $e^{i\theta}$ satisfies

$e^{i\theta}=\cos\theta+i\sin\theta.$ hence

$$\cos\theta=\frac{1}{2}(e^{i\theta}+e^{-i\theta})$$

$$\sin\theta=\frac{1}{2}(e^{i\theta}-e^{-i\theta})$$

From these equation ,it follows by elementary algebra that formulas (5)-(7) can be rewritten (by rewriting each term separately)as

$$c_0+\sum\{c_n e^{inx}+c_{-n}e^{-inx}\} \quad (8)$$

With c_n defined for all integers n by

$$c_n=\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-inx}dx \quad (9)$$

The series in (8) is usually written in the form

$$\sum_{n=-\infty}^{\infty} c_n e^{inx} \quad (10)$$

4. GENERALIZED FOURIER SERIES

The classical theory of Fourier series has undergone extensive generalization during the last two hundred years. For example, Fourier series can be viewed as one aspect of a general theory of orthogonal series expansion. In this section we shall discuss a few of the more celebrated orthogonal series ,such as legendre series ,and wavelet series.[5],[9]

We begin with legendre series. The first two legendre polynomials are defined to be $P_0(x)=1,$ and $P_1(x)=x.$ for $n=2,3,4,...,$ the n^{th} legendre polynomial P_n is defined by the recursion relation

$$nP_n(x)=(2n-2)xP_{n-1}(x)+(n-1)P_{n-2}(x)$$

5. HARMONIC ANALYSIS

Harmonic analysis is b branch of mathematics concerned with the representation of function or signals as the superposition of basics waves, and the study of and generalization of the notions of Fourier series and fourier transforms. In the past two centuries, it has become a vast subject with applications in areas as diverse as signal processing, quantum mechanics, and neuroscience.

The term harmonics originated in physical eigen value problems, to mean waves whose frequencies are integer multiples of one another, as are the frequencies of the harmonics of music notes, but the term has been generalized beyond its original meaning.

Fourier series can be conveniently studied in the context of Hilbert spaces, which provides a connection between harmonic analysis and functional analysis.

In particular ,since the super position principle holds for solutions of a liner homogeneous ordinary differential equation, if such an equation can be solved in the case of a single sinusoid, the solution for an arbitrary function is immediately available by expressing the original function as a Fourier series and then plugging in the solution for each sinusoidal component. In some special cases where the Fourier series can be summed in closed form , this technique can even yield analytic solutions.[6],[7]

5.1 FOURIER SERIES

Harmonic analysis consist of representing a given periodic function in terms of a series of trigonometric terms. It will be found that this can be done in most practical cases, the series so obtained being called a Fourier series. The general form of the Fourier series for $f(t)$ is given by :

$$f(t)=\frac{1}{2}a_0+(a_1\cos\frac{\pi}{T}t+a_2\cos\frac{2\pi}{T}t+a_3\cos\frac{3\pi}{T}t+......)+(b_1\sin\frac{\pi}{T}t+b_2\sin\frac{2\pi}{T}t+b_3\sin\frac{3\pi}{T}t+......$$

$$=\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos\frac{n\pi}{T}t+b_n\sin\frac{n\pi}{T}t) \qquad (11)$$

Where the a`s and b`s are constants which have to be determined for each function. If $f(t)$ is defined analytically then the a`s and b`s can be found by integration, but, if $f(t)$ is only determined by a table of values, then a numerical method must be used.

5.2 THE EXPRESSION FOR a_n AND b_n

It can be shown, firstly by integrating both sides of eq.11 with respect to t over a period that:

$$a_0 = 2\times \text{mean value of } f(t) \text{ over a period} \qquad (12)$$

Also that if we multiply both sides of eq.11 by cos[$(\frac{n\pi}{T})t$] before integrating that:

$a_n = 2\times$ mean value of $f(t)$ cos[$(\frac{n\pi}{T})t$] over a period (13)

And, similarly by first multiplying by sin [$(\frac{n\pi}{T})t$] that:

$b_n = 2\times$ mean value of $f(t)$ sin[$(\frac{n\pi}{T})t$] over a period (14)

If we have the particularly simple case of a function having a period of 2π then eq.11 becomes:

$$f(t)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}(a_n\cos nt+b_n\sin nt) \qquad (15)$$

And the expressions for a_n and b_n become:

$$a_n = 2\times \text{mean value of } f(t) \cos\ nt \text{ over a period} \qquad (16)$$

$$b_n = 2\times \text{mean value of } f(t) \sin\ nt \text{ over a period} \qquad (17)$$

the expression for a_0 being unchanged

5.3 DETERMINATION OF a_n AND b_n

a_n and b_n are determined using eq.12,13,14 or if the period is 2π by eq.12,16.17. In either case we calculate the mean values from a set of equidistant ordinates spread over a complete period. We must take care not to include both end points since only haif of each can be considered as belonging to the period under consideration, the other halves belonging respectively to the previous and subsequent periods.

6. THE PROPOSED METHODOLOGY OF FINDING FOURIER HARMONICS

The term $\sin(\frac{\pi}{T})t$ and $\cos(\frac{\pi}{T})t$ in a Fourier series are called the fundamental terms. The remaining terms, except $\frac{1}{2}a_0$ are called harmonic, for example the term in $\sin(\frac{3\pi}{T})t$ is called a third harmonic. It will usually be found that the coefficients of the higher harmonics become rapidly smaller and it is very seldom necessary to proceed beyond the sixth harmonic.

To find a Fourier series ,as far as the third harmonic, to the represent the periodic function $f(t)$ given by the table(1)

t^o	0	30	60	90	120	150	180	210	240	270	300	330
$f(t)$	1.0	2.4	4.0	4.8	3.2	4.3	4.3	3.1	-2.0	-2.0	-0.8	0.6

Table(1) The given input function $f(t)$

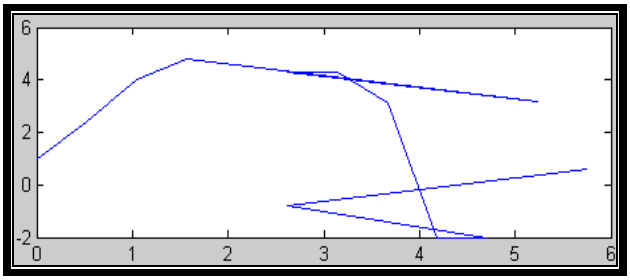
7. SIMULATED OUTPUT RESULTS

We have in this part obtained the best fit for the given data function to Fourier series up to the third harmonic

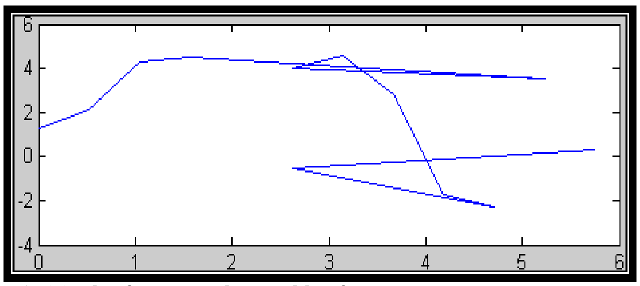
t^o	0	30	60	90	120	150	180	210	240	270	300	330
$f($	1.2	2.1	4.2	4.5	3.5	4.0	4.5	2.9	-1.8	-2.2	-0.6	0.4

Table(2) The function obtained by fitting to Fourier series up to the third harmonic

The figures below explain the relationship between the given input function $f(t)$ and The function obtained by fitting to fourier series up to the third harmonic



Fig(1) The given input function $f(t)$



Fig(2) The function obtained by fitting to Fourier series up to the third harmonic

From the output results, we notice that the obtained values of the function, actual and estimated are almost congruent.

CONCLUSION AND FUTURE WORK

We can use Fourier series for fitting a set of data up to a certain harmonic in a fast and precise way which given us the facility of dealing a certain Fourier expansion rather than the original data points.

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