

## A New Fuzzy Impulsive Control of Chaotic Systems Based on T–S Fuzzy Model



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**Abstract**—In this project, fuzzy impulsive control is used for stabilization of chaotic systems based on the Takagi–Sugeno (T–S) model. The stability issue of the general nonlinear impulsive control system is first investigated via comparison criterion. Then, a novel impulsive control scheme is presented for chaotic systems based on the T–S fuzzy model. Some sufficient conditions are given to stabilize the T–S fuzzy model. The simulation results will be proven to be less conservative theoretically and numerically. Moreover, it is also estimated the stable region of the impulsive interval. Finally, the proposed fuzzy impulsive control scheme is successfully applied to stabilize Rössler’s system and Chua’s circuit. The numerical simulations demonstrate the effectiveness and advantage of main results.

**Keywords**—Impulsive control; MATLAB; Fuzzy;

### I. INTRODUCTION

The Takagi–Sugeno (T–S) fuzzy model is a kind of fuzzy system proposed by Takagi and Sugeno [1], which is described by a set of fuzzy IF–THEN rules to represent local linear input–output relations of a nonlinear system. The main idea of the T–S fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model and to express the overall system by fuzzy “blending” of the local linear system models. As a result, this model can utilize the well established linear system theory to analyze and synthesize a highly nonlinear dynamic systems. A highly nonlinear system can usually be represented by the T–S fuzzy model [1]–[3]. Especially in [4]–[6], the authors have widely used the T–S fuzzy model for the control of chaotic systems. Lian *et al.* represented some well-known chaotic systems by T–S fuzzy models in [7] and designed some locally linear controllers for the synchronization and secure communication of these chaotic fuzzy systems based on a different driving signal. Hence, linear system theory can be easily used to analyze chaotic systems by means of T–S fuzzy models at a certain domain [8], [9]. Manuscript received November 2, 2009; revised May 10, 2010; accepted September 30, 2010. Date of publication November 11, 2010; date of current version April 4, 2011. The work of Y. Liu was supported by the Foundation of the National Natural Science Foundations of China under Grant 10926066, the National Natural Science Foundation of Zhejiang Province under Grant Y6100007, and the Zhejiang Educational Committee under Grant Y200805720. The work of J. Lu was supported by the Natural Science Foundation of Jiangsu Province of China under Grant BK2010408, the Innovation Fund of Basic Scientific Research under Operating

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There are many approaches to analyze the control and synchronization of chaotic systems. In [10], adaptive complete synchronization of two identical or different chaotic (hyperchaotic) systems with fully unknown parameters has been stated. Based on the parallel distributed compensation(PDC) scheme, many results have been obtained for the synchronization of fuzzy systems [11]. The fundamental idea of PDC is that a linear feedback controller is designed for each local linear model. However, there exist some systems which cannot endure continuous control inputs. For these kinds of systems, the PDC-based conventional fuzzy control approach cannot be efficiently utilized. However, an impulsive control technique displayed great efficiency in dealing with this kind of problem. There are many examples where impulsive control can produce better performance than continuous control, and sometimes, even only impulsive approaches can be used [12]. For example, a central bank cannot change its interest rate every day in order to regulate the money supply in a financial market. In fact, impulsive control has arisen in a variety of applications, such as orbital transfer of satellites, ecosystem management, financial modeling, etc. Recently, impulsive control has also gained renewed interest for its promising applications in controlling systems exhibiting chaotic behavior [14]. It was realized that such a control method can stabilize chaotic systems via only small control impulses, while the chaotic behavior may follow unpredictable patterns. In recent years, impulsive systems and impulsive control have been widely studied by researchers [12]–[22]. Especially, it has been widely used to stabilize and synchronize chaotic systems [23], [24]. By integrating the T–S fuzzy model and impulsive control, several authors have proposed a fuzzy impulsive controller to control chaotic dynamical systems [25], [26]. In [27]–[31], the authors have studied impulsive controllers for chaotic systems by using Lyapunov functions, which are required to be non increasing along the whole sequence of the switching. In [32], a unified synchronization criterion for impulsive dynamical networks has been presented.

II. IMPULSIVE CONTROL OF NONLINEAR SYSTEMS

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An impulsive differential system with impulses at fixed moments can be described by

$$\begin{cases} \dot{x}(t) = f(t, x(t)), & t \neq t_k \\ \Delta x(t) = x(t^+) - x(t^-) = U(k, x), & t = t_k, k \in N^+ \end{cases} \quad (1)$$

where  $f: R^+ \times R^n \rightarrow R^n$  is continuous,  $U: R \times R^n \rightarrow R^n$  is continuous,  $x(t) \in R^n$  is the state variable, and  $0 < t_1 < t_2 < \dots < t_k < t_{k+1} < \dots, t_k \rightarrow \infty$  as  $k \rightarrow \infty$ .

Definition 2.1: Let  $V: R^+ \times R^n \rightarrow R^+$ ; then,  $V$  is said to belong to class  $V_0$  if

1)  $V$  is continuous in  $(t_{k-1}, t_k) \times R^n$  and for each  $x \in R^n, k \in N^+, \lim_{(t,y) \rightarrow (t_k^+, x)} V(t, y) = V(t_k^+, x)$  exists;

2)  $V$  is locally Lipschitzian in  $x$ .

Definition 2.2: For  $(t, x) \in (t_{k-1}, t_k) \times R^n, k \in N^+$ , we define

$$D^+V(t, x) = \limsup_{h \rightarrow 0^+} \frac{1}{h} [V(t+h, x+h f(t, x)) - V(t, x)].$$

Definition 2.3: Comparison system: Let  $V \in V_0$ , and assume that

$$\begin{cases} D^+V(t, x) \leq g(t, V(t, x)), t \neq t_k \\ V(t, x + U(k, x)) \leq \psi_k(V(t, x)), t = t_k, k \in N^+ \end{cases}$$

where  $g: R^+ \times R^+ \rightarrow R$  is continuous, and  $\psi_k: R^+ \rightarrow R^+$  is nondecreasing. Then

$$\begin{cases} \dot{w}(t) = g(t, w(t)), t \neq t_k \\ w(t_k^+) = \psi_k(w(t_k)), t = t_k, k \in N^+ \\ w(t_0^+) = w_0 \geq 0 \end{cases} \quad (2)$$

is said to be the comparison system of (1).

III. TAKAGI-SUGENO IMPULSIVE FUZZY MODEL AND IMPULSIVE CONTROL

we will derive our main results concerning T-S impulsive fuzzy model and impulsive control. Consider the following chaotic system:

IV. SIMULATION EXAMPLES

In order to verify the performance of the proposed method, two typical continuous chaotic systems are taken as examples with numerical simulations carried out.

$$\dot{x}(t) = f(x(t)) \quad (4)$$

where  $x(t) \in R^n$  is the state variable  $f \in C[R^+, R^+]$ . The T-S fuzzy model of (4) can be described by fuzzy IF-THEN rules, where the consequent parts represent locally linear models for nonlinear systems. Suppose that T-S fuzzy model is given as follows:

Model rule  $i$ :

IF  $z_1(t)$  is  $M_{i1}$  and  $\dots$  and  $z_p(t)$  is  $M_{ip}$

THEN  $\dot{x}(t) = A_i x(t) + \eta_i(t), \quad i = 1, 2, \dots, r$

where the premise variables  $z_1(t), \dots, z_p(t)$  are proper state variables,  $r$  is the number of the fuzzy rules,  $M_{ij}(j = 1, 2, \dots, p)$  are fuzzy sets,  $A_i$  are system matrices with appropriate dimensions, and  $\eta_i(t)$  are bias terms, which can be time varying. Then, the final output of the fuzzy system can be inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^r h_i(z(t)) \{A_i x(t) + \eta_i(t)\}$$

Rössler's system

The chaotic Rössler's system with input terms is described by

$$\begin{cases} \dot{x}_1(t) = -x_2(t) - x_3(t) + u_1(t) \\ \dot{x}_2(t) = x_1(t) + a x_2(t) + u_2(t) \\ \dot{x}_3(t) = b x_1(t) - (c - x_1(t)) x_3(t) + u_3(t). \end{cases}$$

This system can be exactly represented by a T-S fuzzy model as follows:

Rule 1: IF  $x_1(t)$  is  $F_1(x_1(t))$

THEN  $\dot{x}(t) = A_1 x(t) + u(t)$

Rule 2: IF  $x_1(t)$  is  $F_2(x_1(t))$

THEN  $\dot{x}(t) = A_2 x(t) + u(t)$

where  $x(t) = [x_1(t), x_2(t), x_3(t)]^T$ , and  $u(t) = [u_1(t), u_2(t), u_3(t)]^T$

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -h_1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & h_1 \end{bmatrix}$$

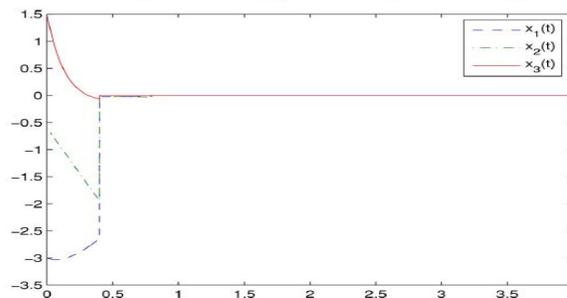


Fig. 1. The simulation of Rössler's system under fuzzy impulsive control with  $\Delta = 0.4$ .

Chua's circuit with input terms is described by

$$\begin{cases} \dot{x}_1(t) = \sigma_1(-x_1(t) + x_2(t) - f(x_1(t))) + u_1(t) \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t) + u_2(t) \\ \dot{x}_3(t) = -\sigma_2 x_2(t) + u_3(t) \end{cases}$$

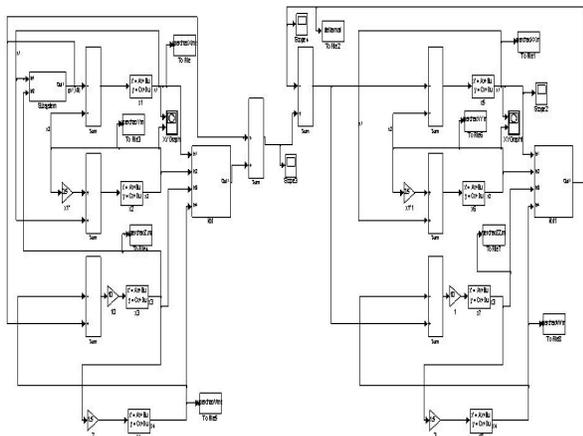


Fig.1:Simulation Model.

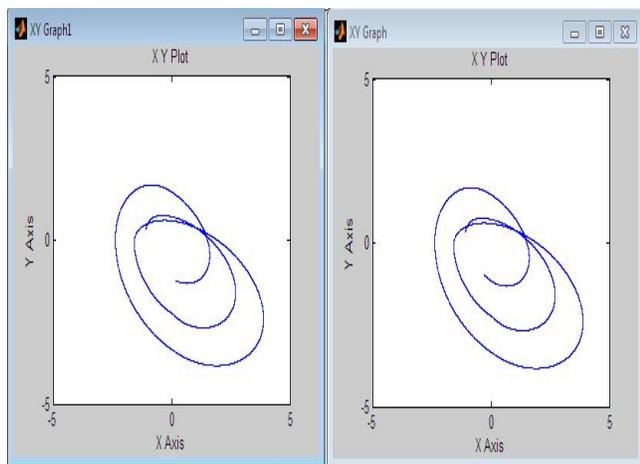


Fig.2: Simulation Outputs

## V. CONCLUSION

The impulsive control technique has been analyzed in the framework of fuzzy systems based on T-S model. According to the general asymptotical stability criteria of impulsive control on nonlinear impulsive systems, we have presented some new fuzzy impulsive criteria to control chaotic dynamical systems. In fact, the obtained fuzzy impulsive criteria can also be used for the synchronization of chaotic systems. Furthermore, the estimation of the stable region of the impulsive interval has been given, which is greater than it was in some existing results. Two simulation results, including Rössler's system and Chua's circuit, have been given to demonstrate the effectiveness of the theoretic results.

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## VII. REFERENCES

- [1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its application to modeling and control," *IEEE Trans. Syst. Man, Cybern.*, vol. 15, no. SMC-15, pp. 116–132, Feb. 1985.
- [2] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. New York: Wiley, 2001.
- [3] K. Tanaka, T. Ikeda, and H. O. Wang, "A unified approach to controlling chaos via an LMI-based fuzzy control system design," *IEEE Trans. Circ. Syst.*, vol. 45, no. 10, pp. 1021–1040, Oct. 1998.
- [4] C. W. Park, C. H. Lee, and M. Park, "Design of an adaptive model based controller for chaotic dynamics in Lorenz systems with uncertainty," *Inf. Sci.*, vol. 147, pp. 245–266, 2002.
- [5] E. Ott, C. Grebogi, and J. A. Yorke, "Controlling chaos," *Phys. Rev. Lett.*, vol. 64, pp. 1196–1199, 1990.
- [6] Q. Gan and C. J. Harris, "Fuzzy local linearization and local basis function expansion in nonlinear system modeling," *IEEE Trans. Syst. Man, Cybern.*, vol. 29, no. 4, pp. 559–565, Aug. 1999.
- [7] K. Y. Lian, C. S. Chiu, T-S. Chiang, and P. Liu, "LMI-based fuzzy chaotic synchronization and communications," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 4, pp. 539–553, Aug. 2001.
- [8] Y. Wang, Z. H. Guan, and H. O. Wang, "Impulsive synchronization for Takagi-Sugeno fuzzy model and its application to continuous chaotic system," *Phys. Lett. A*, vol. 339, pp. 325–332, 2005.
- [9] Y. Wang, Z. H. Guan, H. O. Wang, and J. Xiao, "Impulsive control for T-S fuzzy system and its application to chaotic systems," *Int. J. Bifurcat. Chaos*, vol. 16, pp. 2417–2423, 2006.
- [10] J. Q. Lu, and J. D. Cao, "Adaptive complete synchronization of two identical or different chaotic (hyperchaotic) systems with fully unknown parameters," *Chaos*, vol. 15, no. 4, p. 043901-1–043901-10, 2005.
- [11] H. O. Wang, K. Tanaka, and M. Griffin, "An approach to fuzzy control of nonlinear systems: Stability and design issues," *IEEE Trans. Fuzzy Syst.*, vol. 4, no. 1, pp. 14–23, Feb. 1996.
- [12] T. Yang, *Impulsive Control Theory*. Berlin/Heidelberg, Germany: Springer, 2001.
- [13] T. Yang, L. B. Yang, and C. M. Yang, "Impulsive control of Lorenz system," *Physica D*, vol. 110, pp. 18–24, 1997.
- [14] T. Yang and L. O. Chua, "Impulsive stabilization for control and synchronization of chaotic systems: Theory and

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- application to secure communication," *IEEE Trans. Circ. Syst.*, vol. 44, no. 10, pp. 976–988, Oct. 1997.
- [15] V. Lakshmikantham, D. D. Bainov, and P. S. Simeonov, *Theory of Impulsive Differential Equations*. Singapore: World Scientific, 1989.
- [16] Z. G. Li, C. Y. Wen, and Y. C. Soh, "Analysis and design of impulsive control systems," *IEEE Trans. Automat. Control*, vol. 46, no. 6, pp. 894– 897, Jun. 2001.
- [17] M. Itoh, T. Yang, and L. O. Chua, "Conditions for impulsive synchronization of chaotic and hyperchaotic systems," *Int. J. Bifurcat. Chaos*, vol. 11, pp. 551–560, 2001.
- [18] Y. Liu and S. W. Zhao, "A new approach for the impulsive stabilization of Liu's system," *Commun. Theor. Phys.*, vol. 5, pp. 994–998, 2010.
- [19] Y. Liu and S.W. Zhao, "A new approach to practical stability of impulsive functional differential equations in terms of two measures," *J. Comput. Appl. Math.*, vol. 223, pp. 449–458, 2009.
- [20] J. T. Sun, Y. P. Zhang, and Q. D. Wu, "Less conservative conditions for asymptotic stability of impulsive control systems," *IEEE Trans. Automat. Control*, vol. 48, no. 5, pp. 829–831, May 2003.
- [21] S. W. Zhao and J. T. Sun, "A Lie algebraic condition of stability for hybrid systems and application to hybrid synchronization," *Int. J. Bifurcat. Chaos*, vol. 19, pp. 379–386, 2009.
- [22] Y. W. Wang, Z. H. Guan, and J. W. Xiao, "Impulsive control for synchronization of a class of continuous systems," *Chaos*, vol. 14, pp. 199–203, 2004.
- [23] X. Z. Liu, "Impulsive stabilization and control of chaotic system," *Nonlinear Anal.*, vol. 47, pp. 1081–1092, 2001.
- [24] C. Li and X. Liao, "Complete and lag synchronization of hyperchaotic systems using small impulsive," *Chaos, Sol. Fract.*, vol. 22, pp. 857–867, 2004.
- [25] Y. B. Nian, and Y. A. Zheng, "Synchronization for Rössler chaotic systems using fuzzy impulsive controls," in *Proc. IHHMSP*, 2007, pp. 179–182.
- [26] Y. A. Zheng and G. R. Chen, "Fuzzy impulsive control of chaotic systems based on T-S fuzzy model," *Chaos, Sol. Fract.*, vol. 39, pp. 2002–2011, 2009.
- [27] Q. S. Zhong, J. F. Bao, Y. B. Yu, and X. F. Liao, "Impulsive control for T-S fuzzy model-based chaotic systems," *Math. Comp. Sim.*, vol. 79, pp. 409–415, 2008.
- [28] Q. S. Zhong, J. F. Bao, and Y. B. Yu, "Impulsive control for T-S fuzzy model based time-delay chaotic systems", in *Proc. ICCAS*, 2008, pp. 987–990.
- [29] X. W. Liu and S. M. Zhong, "T-S fuzzy model-based impulsive control of chaotic systems with exponential decay rate," *Phys. Lett. A*, vol. 370, pp. 260–264, 2007.
- [30] X. H. Zhang, D. Li, and G. Yang, "Impulsive control of T-S fuzzy systems," in *Proc. FSKD*, 2007, pp. 321–325.
- [31] X. H. Zhang, A. Khadra, D. Li, and D. Yang, "Impulsive stability of chaotic systems represented by T-S model," *Chaos Sol. Fract.*, vol. 41, pp. 1863–1869, 2009.
- [32] J. Q. Lu, D.W. C. Ho, and J. D. Cao, "A unified synchronization criterion for impulsive dynamical networks," *Automatica*, vol. 46, pp. 1215–1221, 2010.