



Finite Difference Approximation Method Of Biharmonic Equation In Human Face Recognition

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ABSTRACT

This paper addressed the problem of Biharmonic equation in human face recognition field. PDE has been treated numerically to get the discrete quantity values (Matrices) using finite difference method. Three matrices in different point number are used with human face image in order to match the plot intensity image.

Keywords: Biharmonic equation, Face recognition, Finite difference method, Intensity image, Matrices.

1. INTRODUCTION

The use of Biometrics has been proposed as a solution to these many needs. Biometrics is physical characteristics or personal traits of a person that can be measured, and be used to recognize the person either by identification or verification. Identification occurs when the biometric system identifies a person from the whole enrolled population by searching a database for a match. This process is sometimes called "one-to-many" matching. Verification occurs when the biometric system authenticates a person's claimed identity from his previously enrolled pattern. This called "one-to-one" matching [1].

In the field of computer vision, researchers have proposed many techniques for representation and analysis of the varying shape of objects, such as active contour and deformable template. In the field of pattern recognition, researchers have focused on the analysis of texture patterns of the face image and have used many kinds of pattern analysis methods such as principal component

analysis (PCA) independent component analysis and wavelet decomposition. Although there were some methods that considered the shape variation of the face such as the Gabor wavelet jets, they concentrated on the analysis of holistic face texture assuming they are deal with only front view face image. In the modern life, the need for personal security and access control is becoming an important issue. Biometrics is the technology which is expected to replace traditional authentication methods that are easily stolen, forgotten and duplicated. Fingerprints, face, iris, and voiceprints are commonly used biometric features. Among these features, face provides a more direct, friendly and convenient identification method and is more acceptable compared with the individual identification methods of other biometrics features. Thus, face recognition is one of the most important parts in biometrics [2].

2. DIGITAL IMAGE IDEA

An image may be defined as a two dimensional function $U(x, y)$ where x and y are spatial (plane) coordinates and the amplitude of f at any pair of coordinates (x, y) is called the intensity or gray level of the image at that point. When x, y and the amplitude values of f are all finite, discrete quantities, we call the image digital image. The digital image is composed of a finite number of elements, each of which has a particular location and values. These elements are referring to as picture elements called pixel [4]. The field of digital image processing refers to processing digital images by means of digital computer.

3. GRAY LEVEL REPRESENTATION

The principle energy sources for images in use today is the electromagnetic energy spectrum, so we use an image and present how can convert the image to digital image or numerical, in order to be useful for computer language. The image is quantities of energy which scaling from white to black and any energy level distinguish by integer number from 0 to 255 which represented all colors in the images started from white colors which take digital integer number zero to 255 respected to black colors corresponding with electromagnetic energy spectrum levels as show in Figure (1) [3].

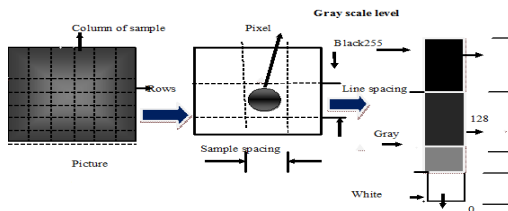


Figure 1: Electromagnetic energy spectrum levels or gray level [3].

The image is continuous function; to digitize the image (discrete quantities) we use numerical divided method .

4. BIHARMONIC EQUATION METHODS IN FACE RECOGNITION FIELD

The analytical and numerical are two major methods of Biharmonic equation (4-order Laplace Equation) in field of face recognition, the purpose of PDE is used to generate complex geometric shape based on a suitably boundary condition for the Partial Differential Equation. It's well known that the Biharmonic Equation is Elliptic Equation, which generates the Elliptic Surface in order to contain the face information and the Laplace operator grantee the smooth transition between the boundaries. We use the Equation (1) numerically in recognition filed in order to measure the efficiency of this method in recognition purpose.

$$(\nabla^2)^2 U = \frac{\partial^4 U}{\partial x^4} + 2 \frac{\partial^4 U}{\partial x^2 \partial y^2} + \frac{\partial^4 U}{\partial y^4} = 0 \tag{1}$$

5. NUMERICAL SOLUTION OF BIHARMONIC EQUATION

We can simply write down its discrete finite difference approximation for a given

neighborhood of data points. Suppose that for example, for the Laplace operator, by denoting a node in the square finite difference mesh by the integers i, j , with mesh spacing h so that the coordinates $u = ih$ and $v = jh$, let $k = h = 1$, so we have the Biharmonic equation based on Laplace in four orders (Elliptic Equation) .

The discrete quantities of equation 1 by using forward finite difference method as follow

$$\begin{aligned} &= U(x_{i+2h}, y_j) - 8U(x_{i+h}, y_j) + 20U(x_i, y_j) \\ &- 8U(x_{i-h}, y_j) + U(x_{i-2h}, y_j) + U(x_i, y_{j+2k}) \\ &- 8U(x_i, y_{j+k}) - 8U(x_i, y_{j-k}) \\ &+ U(x_i, y_{j-2k}) + 2U(x_{i+h}, y_{j+k}) \\ &+ 2U(x_{i-h}, y_{j-k}) + 2U(x_{i+h}, y_{j-k}) \\ &+ 2U(x_{i-h}, y_{j+k}) \\ &= 0 \end{aligned} \tag{5}$$

The finite difference scheme for Biharmonic Equation can be represented by the 5- point stencil shown in Figure 2.

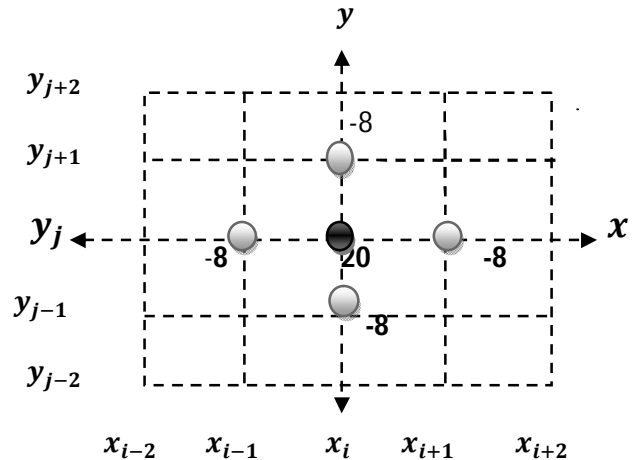


Figure 2: The 5-points stencil Biharmonic Equation

The finite difference scheme for Biharmonic Equation can be represented by the 9- point stencil shown in Figure 3.

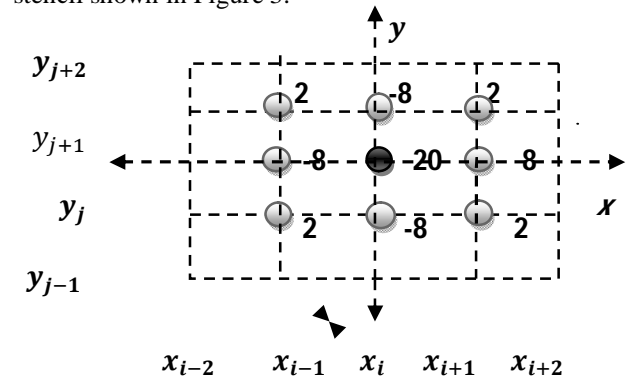


Figure 3: The 9-points stencil of Bioharmonic Equation

The finite difference scheme for Biharmonic Equation can be represented by the 13- point stencil shown in Figure 4.

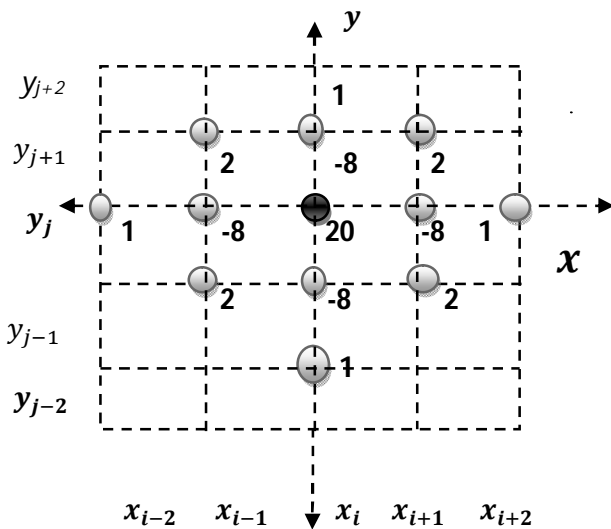


Figure 4: The 13-point's stencil of Biharmonic Equation

6. RESULTS

In this section we discuss the comparison between the intensity of original human face image and the combine human face image with matrices of Bioharmonic Equation in different there points.

Result 1: the intensity and plotting of original human face image



Figure 5a: Original face image intensity

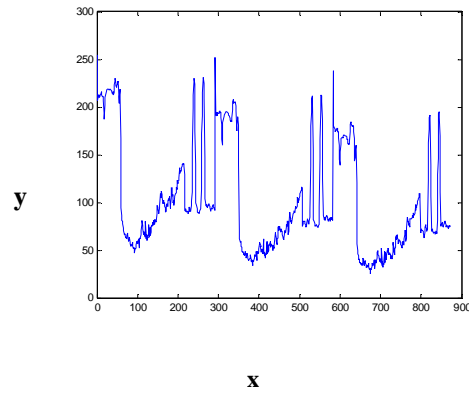


Figure 5b: Corresponding image intensity plot of Figure 5a

Result 2: The intensity and plotting of image with 5 point matrix.



Figure 6a: Intensity of face image with 5-point matrix.

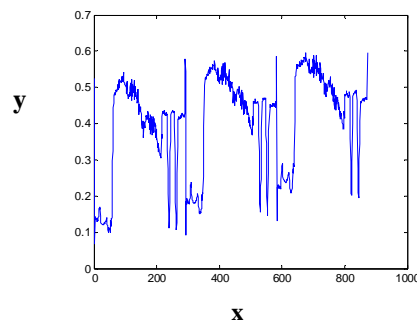


Figure 6b: Corresponding image intensity plot of Figure 6a.

Fig (5-b): Corresponding image intensity

plot

Result 3: The intensity and plotting of image with 9-point matrix.



Figure 7a: Intensity of face image with 9-point matrix.

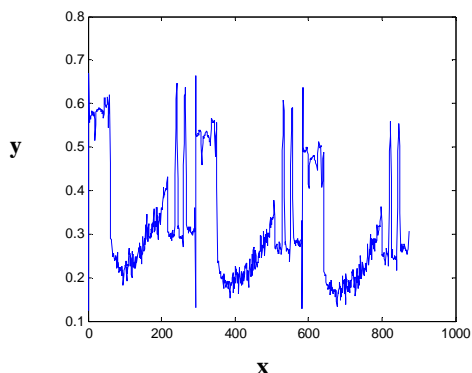


Figure 7b: Corresponding image intensity plot of Figure 7a.

Result 4: The intensity and plotting of image with 13-point matrix.



Figure 8a: Intensity of face image with 13-point matrix.

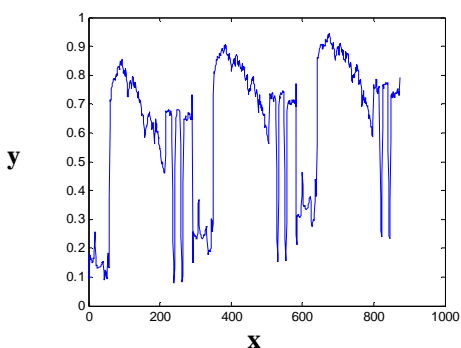


Figure 8b: Corresponding image intensity plot of Figure 8a.

As the result mentioned above, the effect of the number of points in discrete method, it's clearly. So that the Figure 7a and Figure 7b are closed as original image figure in spite of approximate solution of Biharmonic Equation, also the effect of points $U(x_{i+h}, y_{j+k}) = U(0,0)$ in Figure 6 and 8. In Figure 8a and 8b the coefficient of central point $U(x_i, y_j)$ represented as 1, because the plot of exact coefficient of $U(x_i, y_j)$ is invisible.

7. CONCLUSION

In this paper we have applied the technique of numerical method in field of human face recognition. We utilize 5-point, 9-point and 13-point, whereby the explicit finite difference method solution of standard Biharmonic Equation is solved for appropriately approximation points We applied algorithm which mentioned in [5] with MATLAB. The work has potential for developing software tools, whereby the numerical solution of PDE could be used to efficiency represent and plot the intensity of human face and compare with the original human face image. Further work on this theme we need to refine the solution of Biharmonic Equation in order to investigate the possibility of using PDE in human filed recognition.

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